DARK MATTER HALOS

or nonlinear growth

SPHERICAL COLLAPSE MODELS

- Once perturbations become large, $\delta > 1$, they do not evolve independently nor follow our simple linear theory. In order to follow the density evolution requires numerical techniques (N-body simulations).
- However, we can gain insight into what will happen by considering high degrees of symmetry, like the collapse of a region with perfect spherical symmetry.
- These types of models can be useful in giving us insight into what occurs and the limitations of our physical understanding.

SPHERICAL COLLAPSE IN A Λ=0 UNIVERSE

- In the absence of dark energy a shell in a spherical density perturbation evolves according to

$$\frac{d^2r}{dt^2} = -\frac{GM}{r^2}$$

- This can be integrated once to give

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 - \frac{GM}{r} = \mathcal{E}$$

- where $\mathscr{E}$ is the specific energy of the shell. For $\mathscr{E} < 1$ collapsing shells the motion of the shell can be written as

$$r = A(1 - \cos(\theta)) \qquad t = B(\theta - \sin(\theta))$$

SPHERICAL COLLAPSE IN A Λ=0 UNIVERSE

$$r = A(1 - \cos(\theta)) \qquad t = B(\theta - \sin(\theta))$$

- We can see from these functions that the maximum radius the shell will reach is r_{max} = 2A at a time $t_{max}=\pi B$.
- We also see that the shell would reach the center, r=0 at t_{col} = $2t_{max}$. While this isn't really physical, conceptually we define two times the time where the shell reaches maximum expansion as the time when the shell collapses.
- We can ask what would the overdensity be assuming just linear growth until this time and get

$$\delta_c(t_{col}) = \frac{3D(t)}{5(1+z)}\left[\left(\frac{\pi}{\Omega_m^{1/2}(t_{col})H(t_{col})t_{col}}\right)^{2/3} - \left(1 - \Omega_m^{-1}(t_{col})\right)\right] \approx 1.686(\Omega_m(t_{col}))^{0.0185}$$

SPHERICAL COLLAPSE IN A Λ>0 UNIVERSE

- With dark energy the equation for a shell becomes

$$\frac{1}{2}\left(\frac{dr}{dt}\right)^2 - \frac{GM}{r} - \frac{\Lambda c^2}{6} r^2 = \mathcal{E}$$

- this is harder to solve in closed form, but with a number of approximations one can get the overdensity and the collapse time as

$$\delta_c(t_{col}) = \frac{3D(t)}{5(1+z)}\left(1 + \frac{\Lambda c^2 r_{max}^2}{6GM}\right)\left(\frac{6GM\Omega_\Lambda(t_{col})}{\Lambda c^2 r_{max}^2 \Omega_m(t_{col})}\right)^{1/3} \approx 1.686(\Omega_m(t_{col}))^{0.0055}$$

COLLAPSE OVERDENSITY

- In both cases the dependence on Ω_m is very weak (0.0185 and 0.0055) so we can take the linear density when collapse occurs to be $\delta_c = 1.686$.
- Note that this is the density a perturbation would have extrapolated from linear theory, where it clearly doesn't apply. Alternatively, the density would be infinite in the shell model when r=0.
- This model gives us a sense of the linear overdensity that would be a collapsed object. Which is how we will use it.

VIRIAL THEOREM

- What would be the size or density of our collapsed objects?

- We can use the virial theorem to relate the kinetic and potential energies of our collapsed halo.

$$2 < KE >= - < PE >$$

- Let's consider the collapse of a uniform sphere of mass M in a Λ=0 universe. At maximum expansion the kinetic energy is zero so the total energy would be

$$E = -\frac{3GM^2}{5r_{max}}$$

VIRIAL DENSITY

- There is no dissipation of energy so after collapse the total energy is the same but now PE = 2E. Since the mass in the system doesn't change the radius after collapse would be, $r_{vir} = 1/2\ r_{max}$.

- The mean overdensity within r_{vir} at t_{vir} is then

$$1 + \Delta_{vir} = \frac{\rho(t_{max})}{\bar{\rho}(t_{vir})}\left(\frac{r_{max}}{r_{vir}}\right)^3$$

- where $\bar{\rho}$ is the mean matter density of the universe.

- For $\Omega_m = 1$ we get $\Delta_{vir} = 18\pi^2 \cong 178$.

VIRIAL DENSITY

- For open or flat universes with dark energy the situation is much more complicated. Bryan & Norman (1998) provided fitting functions where $x = \Omega_m(t_{vir}) - 1$.
  - Open - $$\Delta_{vir} \approx \frac{18\pi^2 + 60x - 32x^2}{\Omega_m(t_{vir})}$$
  - Flat - $$\Delta_{vir} \approx \frac{18\pi^2 + 82x - 39x^2}{\Omega_m(t_{vir})}$$

IDENTIFYING DARK MATTER HALOS

- These definitions are often used in defining dark matter halos. Note that in reality there is only a density field, it has no edges.
- But conceptually it is convenient to consider a dark matter halo as an object that can be associated with a galaxy.
- There are two main ways this is done:
  - One is to use the Friends-of-Friends algorithm which associates particles into a group by a linking length. The problem with this method is that the shape of these groups can be anything and the choice of liking length is basically arbitrary.
  - Another choice is to consider when the spherical overdensity in some region has a particular value. That value is often set by the virial overdensity definition we have just discussed. Though people also just take $200\rho_c$ as an approximation to that.

PEAKS

- Clearly halos should form where there are peaks in the density field.   In order to normalize out the power spectrum we can normalize the peak height by the mass variance.

$$\nu = \frac{\delta}{\sigma}$$

- The statistical properties of peaks can be calculated analytically for Gaussian random fields. Thus one can determine the number density, ellipticity, and correlation function for peaks analytically.

HALO MASS FUNCTION

- One thing it would definitely be interesting to know is the number of halos of different masses. An analytic model using the spherical collapse model was worked out for this by Press & Schechter in 1974.

- They considered smoothing the density field by a window function

$$\delta_s(x:R) = \int \delta_i(x')W(x+x':R)d^3x'$$

- Then they reasoned that the probability that $\delta_s > \delta_c(t)$ is the same as the number of mass elements that at time t are in halos with masses greater than M. If δ_i is a Gaussian random filed then so is δ_s and the probability is given by

$$P[\delta_s > \delta_c(t)] = \frac{1}{2\pi\sigma(M)} \int_{\delta_c(t)}^{\infty} \exp\left(-\frac{\delta_s^2}{2\sigma^2(M)}\right) d\delta_s = \frac{1}{2}\mathrm{erfc}(\delta_s)$$

PRESS-SCHECHTER MASS FUNCTION

$$P[\delta_s > \delta_c(t)] = \frac{1}{2\pi\sigma(M)}\int_{\delta_c(t)}^{\infty} \exp\left(-\frac{\delta_s^2}{2\sigma^2(M)}\right) d\delta_s = \frac{1}{2}\mathrm{erfc}(\delta_s)$$

- According to the argument the number of halos with mass greater than M should just be the same thing. But as M ➞ 0 the probability function goes to 1/2. Which makes sense because half of the density fluctuations are negative. Press & Schechter multiplied this by 2 so that all mass is in halos. That gives a differential mass function of

$$n(M,t)dM = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}\delta_c}{M^2\sigma(M)} \exp\left(-\frac{\delta_c^2}{2\sigma^2(M)}\right)\left|\frac{d\ln\sigma}{d\ln M}\right| dM$$

PRESS-SCHECHTER MASS FUNCTION

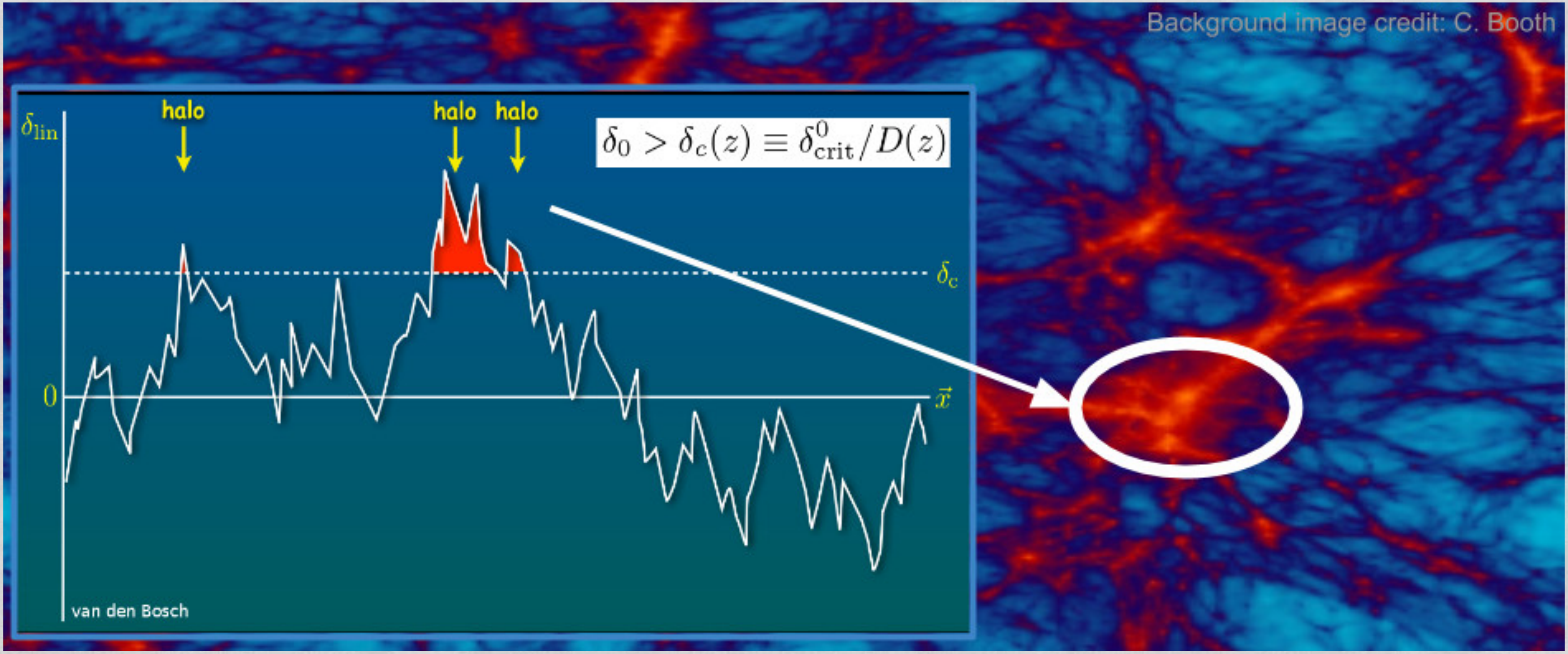
$$n(M,t)dM = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}\delta_c}{M^2\sigma(M)}\exp\left(-\frac{\delta_c^2}{2\sigma^2(M)}\right)\left|\frac{d\ln\sigma}{d\ln M}\right|dM$$

- This is basically a power law for low masses with an exponential cutoff for high masses.
- The transition occurs at a halo mass M* given by

$$\sigma(M*) = \delta_c(t) = \frac{\delta_c}{D(t)}$$

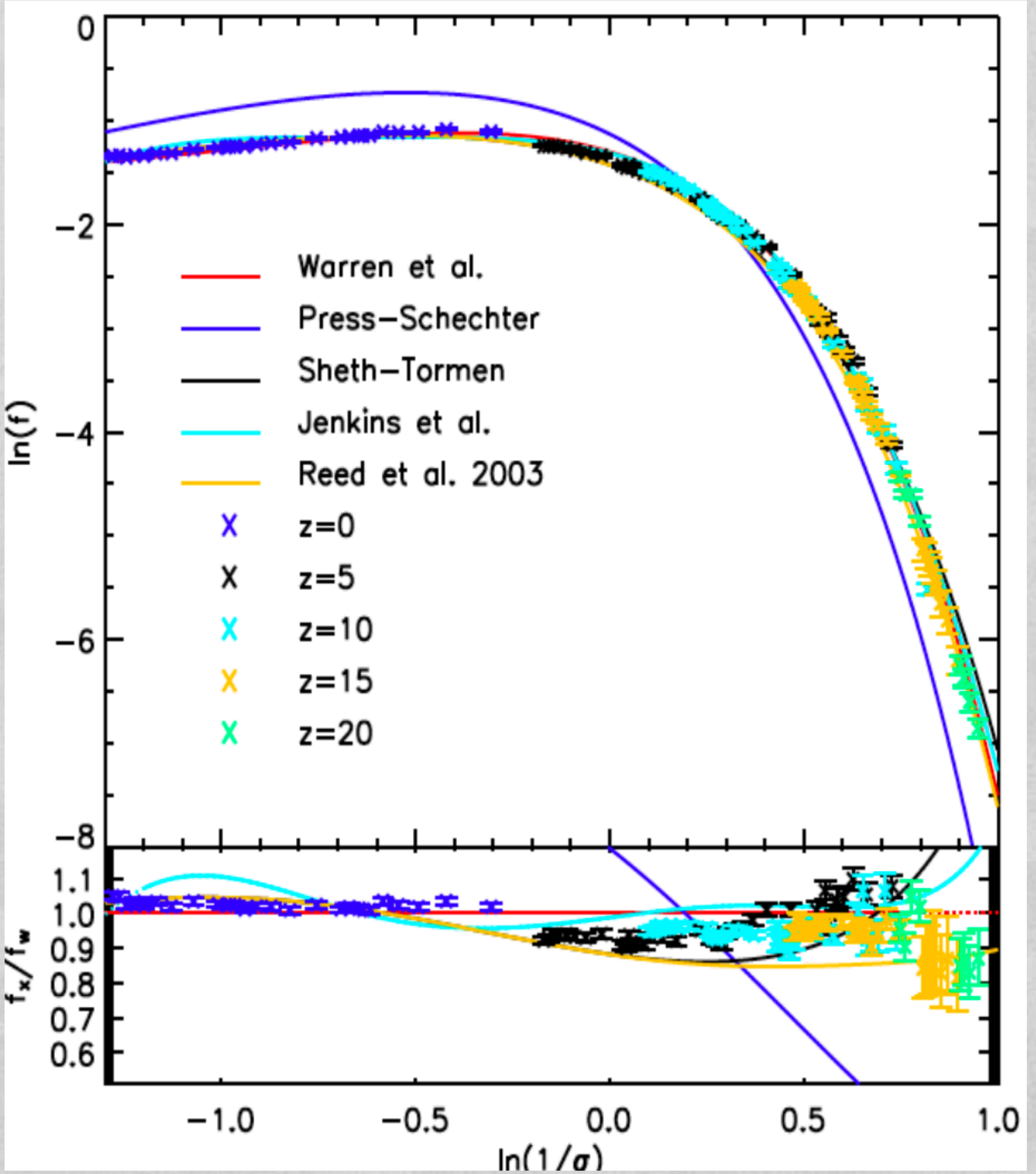
- Schechter also proposed that a similar shape function should fit the luminosity distribution of galaxies

$$n(L)dL = \phi^*\left(\frac{L}{L^*}\right)^\alpha e^{-L/L^*}\frac{dL}{L^*}$$


Background image credit: C. Booth
halo
halo
halo
$\delta_{\rm lin}$
$\delta_0 > \delta_c(z) \equiv \delta^0_{\rm crit}/D(z)$
δ_c
0
$\vec{x}$
van den Bosch

HALO MASS FUNCTION

- Comparing to simulations we see the Press-Schechter mass function is roughly right though differs by factors of a few.
- An improvement by Sheth & Tormen is to consider ellipsoidal instead of spherical collapse. This can give an excellent fit, but requires free parameters.
- Today N-body simulations are used to get the 'correct' results, but the Press-Schechter approach is often used as a conceptual framework.



MERGER HISTORIES

- In the spherical halo model the halo grows by continually accreting new shells of matter, we could call this *smooth accretion*.
- But in reality there are many peaks in the density field. As they grow they attract one another and many will eventually merge.
- This might not matter as far as the total mass of the halo goes, but if we think there are galaxies in the halos then merging halos would lead to merging galaxies which might be very different than smoothly accreted matter.

EXTENDED PRESS SCHECHTER

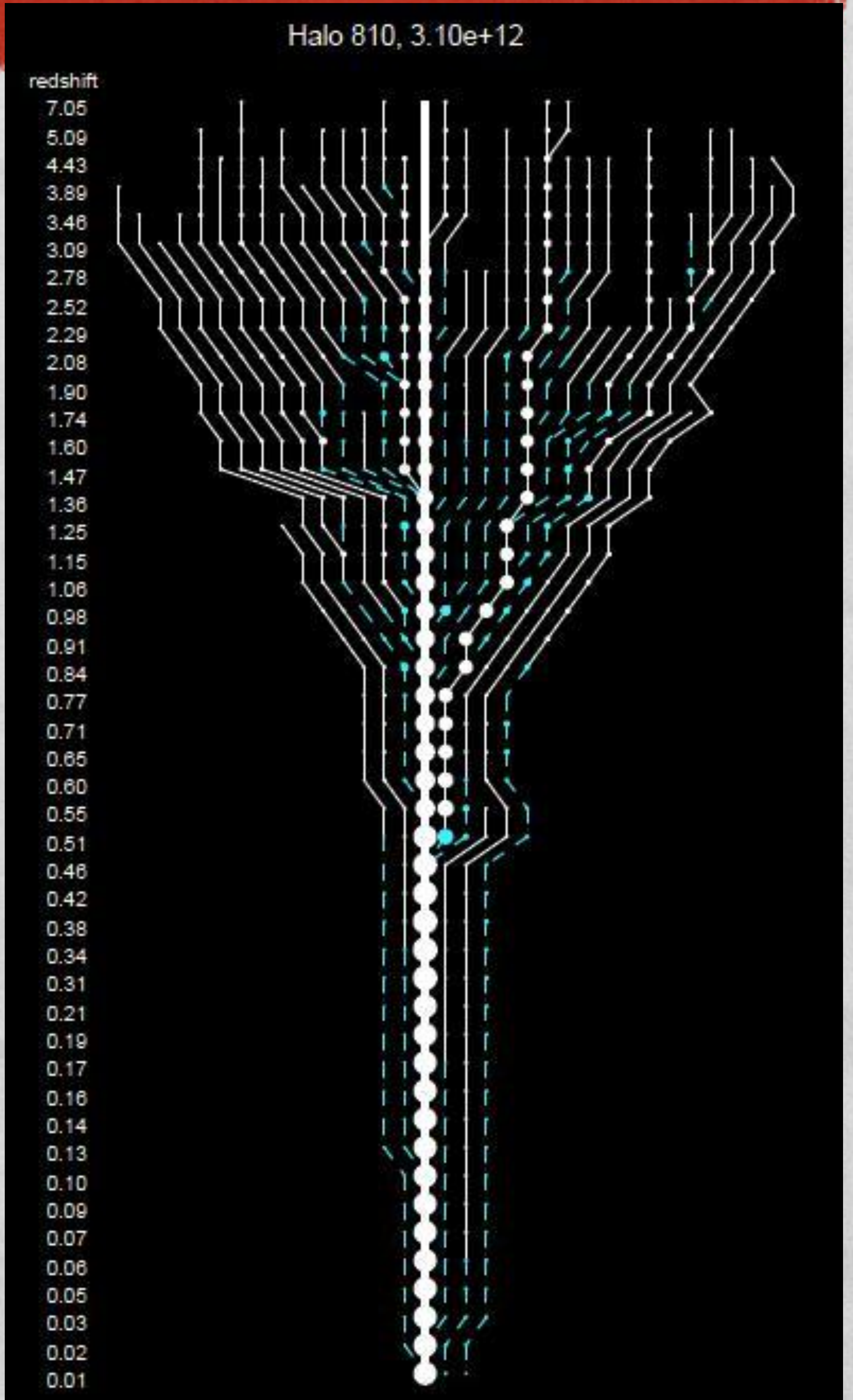
- We can get a sense of how halos merge by extending the Press-Schechter formalism. Remember Press-Schechter asked if the smoothed overdensity in a region was above $\delta_c(t)$.
- Now what if we asked at earlier time if subparts of that region were above $\delta_c(t)$, that would tell us which of those subparts were halos. As a function of time this would give us a halos merging history.
- Since the density of a Gaussian random process the smoothed density essentially does a random walk as we increase the scale. This gives us a way to trace the halos history.

EXTENDED PRESS SCHECHTER

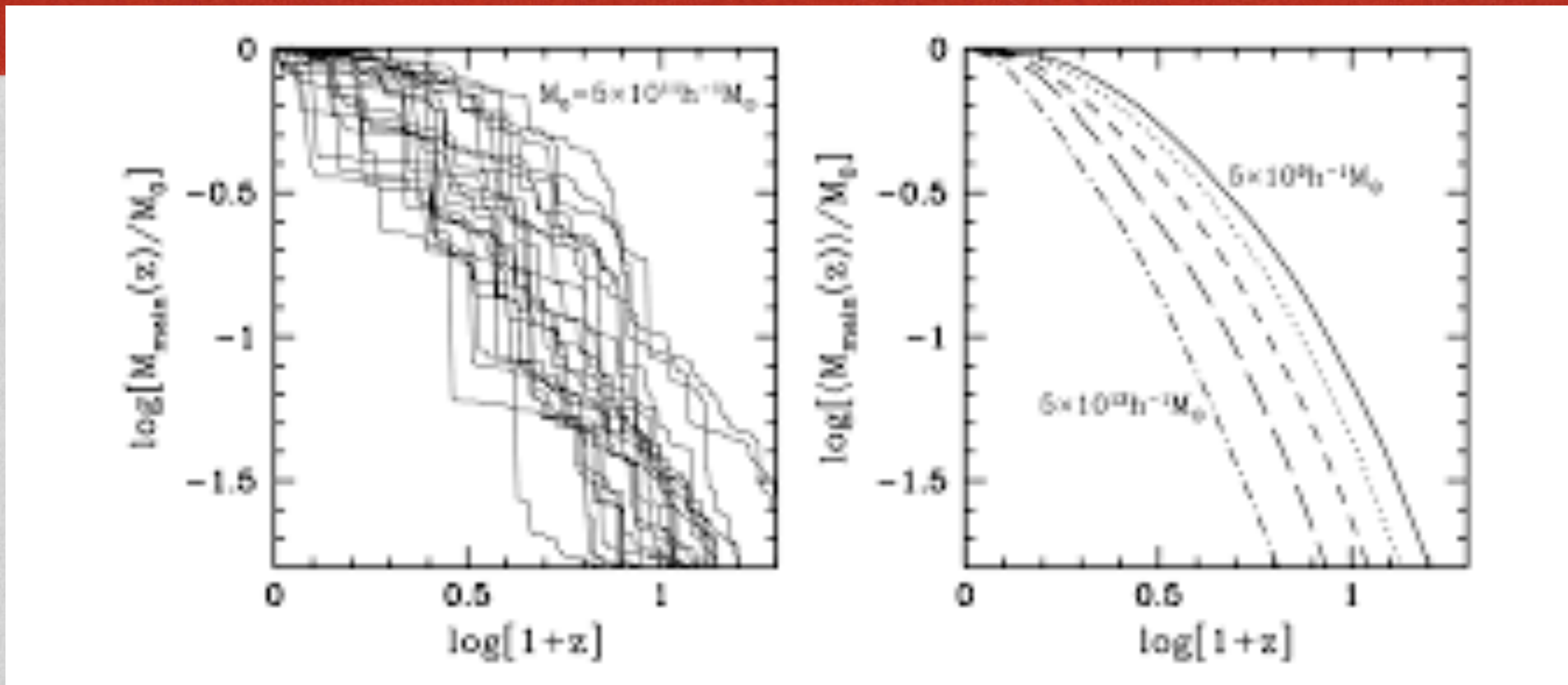
Trajectory B' mirrors trajectory B and therefore is equally likely.

MERGER TREES

- While merger histories can be determined from EPS, it is much more reliable to extract them from N-body simulations (consistant-trees).
- However these will always be resolution dependent, only including halos above a certain mass.
- Merger trees are complicated and hard to work with, often people just focus on the main progenitor branch.



FORMATION TIME



- Many growth histories can be averaged to show how growth depends on mass.
- Lower mass halos reached the same fraction of their current mass a longer time ago.

DENSITY PROFILE OF HALOS

- A simple model of the density profile of a dark matter halo is the isothermal sphere which has the advantage that it gives a flat rotation curve.

$$\rho(r) = \frac{V_h^2}{4\pi G r^2} \qquad M(r) = \frac{V_h^2}{G} r \qquad V(r) = \sqrt{\frac{GM(r)}{r}} = V_h$$

- But the density at r=0 is infinite and the mass enclosed goes to infinity with r.

- Fits to N-body simulations find profiles like the one proposed by Navarro, Frenk and White (NFW) 1996.

$$\rho(r) = \frac{4\rho_s}{(r/r_s)(1 + r/r_s)^2}$$

NFW PROFILE

- This profile goes like r^{-1} at small r and r^{-3} and large r. The halo is truncated at some r_{vir}, because there really is not an edge to a halo it is just a peak in the density field.

- Other studies have found that the inner profile may be as steep as $r^{-1.5}$, but there is disagreement about this. The range from -1 to -1.5 is now generally accepted.

- The ratio $c=r_s/r_{vir}$ is called the halo's concentration as for halos of the same mass it determines if there is more or less mass in the inner parts of the halo.

- The mean concentration of halos has been found to be a function of halo mass with less massive halos being more concentrated. This is generally understood as the concentration having to do with the formation time of the halo, halos that form earlier should be denser and we have shown that formation time is earlier for lower mass halos.

ANGULAR MOMENTUM

- Another interesting property of halos is their angular momentum. The angular momentum can be described by a dimensionless parameter called spin (Peebles 1969) which is basically the square root of the rotational energy compared to the total energy of the system.

$$\lambda = \frac{J|E|^{1/2}}{GM^{5/2}}$$

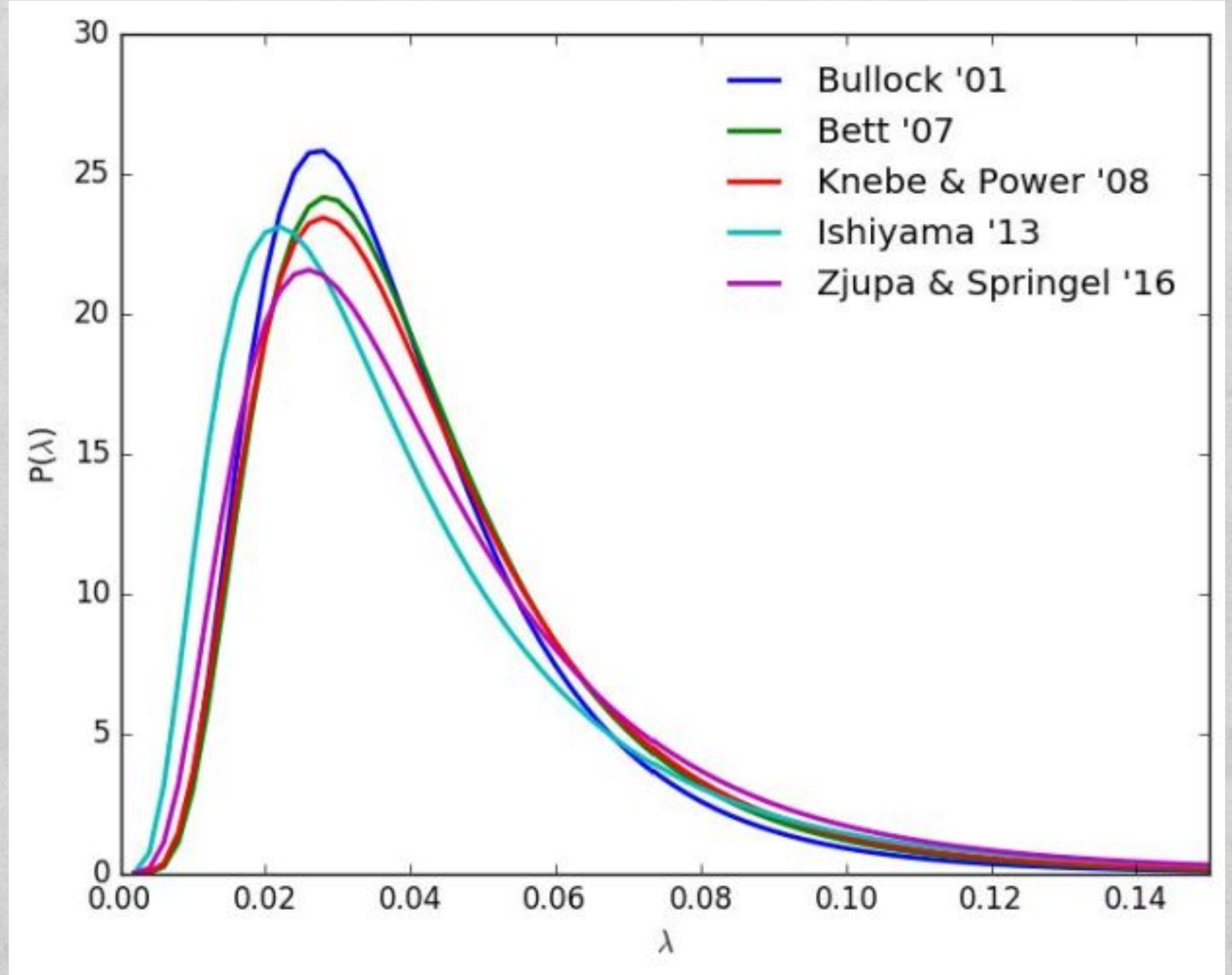
- An alternative definition which avoids the difficulty of calculating the energy of the system and allows for considerations besides the total system

$$\lambda_B = \frac{J}{\sqrt{2}M_{vir}V_{vir}R_{vir}}$$

- The $\sqrt{2}$ makes the values equal for an isothermal sphere.

ANGULAR MOMENTUM

- It has been found that the spin distribution of halo follows a log normal distribution with a mean around λ~0.035.



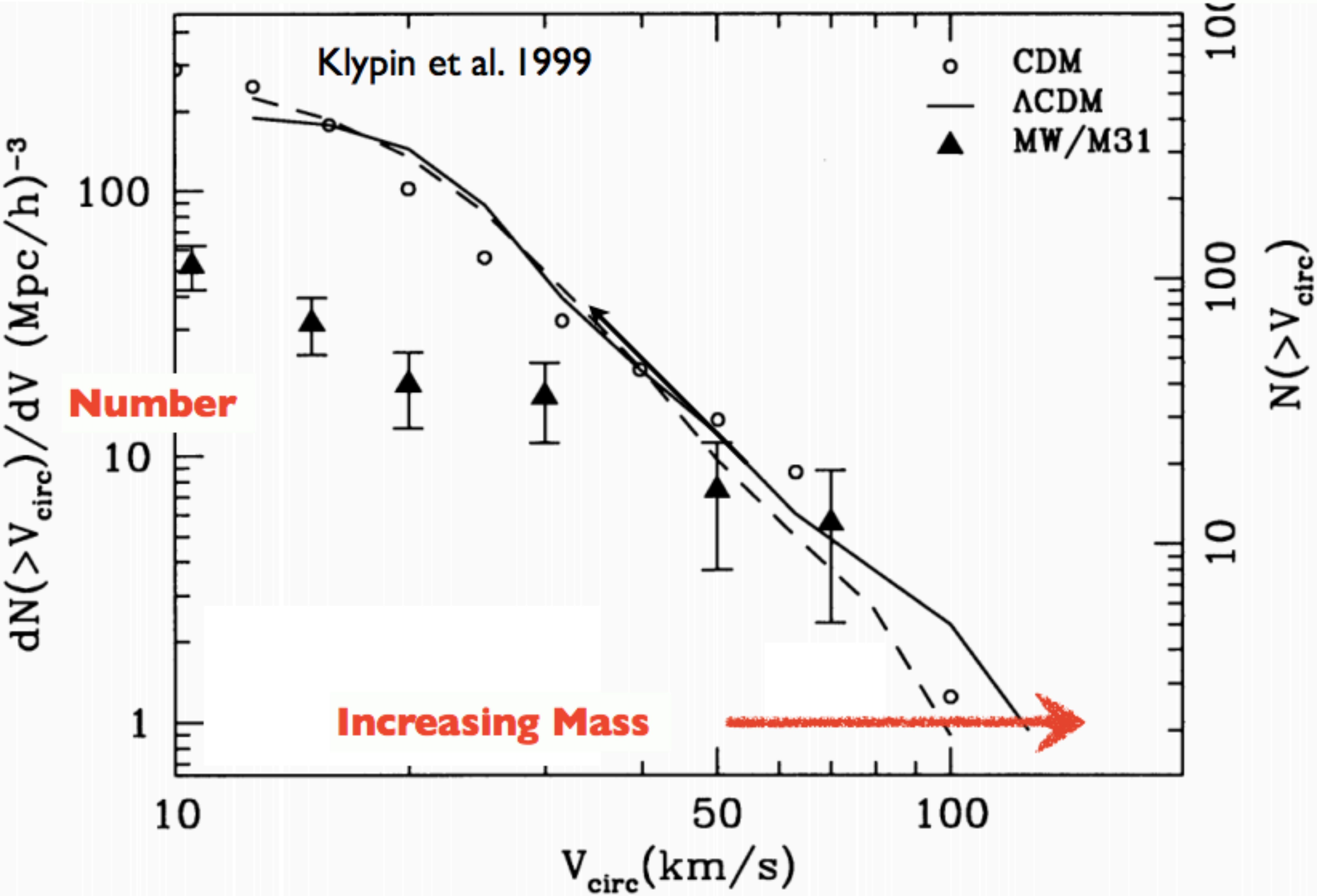
SUBHALOS

- As the resolution of simulations improved they became capable of resolving substructure in a halo.
- Some of this substructure can be identified as sub halos, though definitions but be different. Subhalos are identified by having a density higher than the host halo at that radius.

SUBHALOS

- Resolving subhalos is very tricky and one must be careful considering numerical effects when looking at their destruction and evolution when they get small.
- Measuring the number of sub halos in halos that would host a Milky Way has led to an issue called the missing satellites problem which notes that there are way more sub halos in a simulation then satellite galaxies observed around the Milky Way or Andromeda.

A quantitative comparison of # satellites at r < 400 kpc.

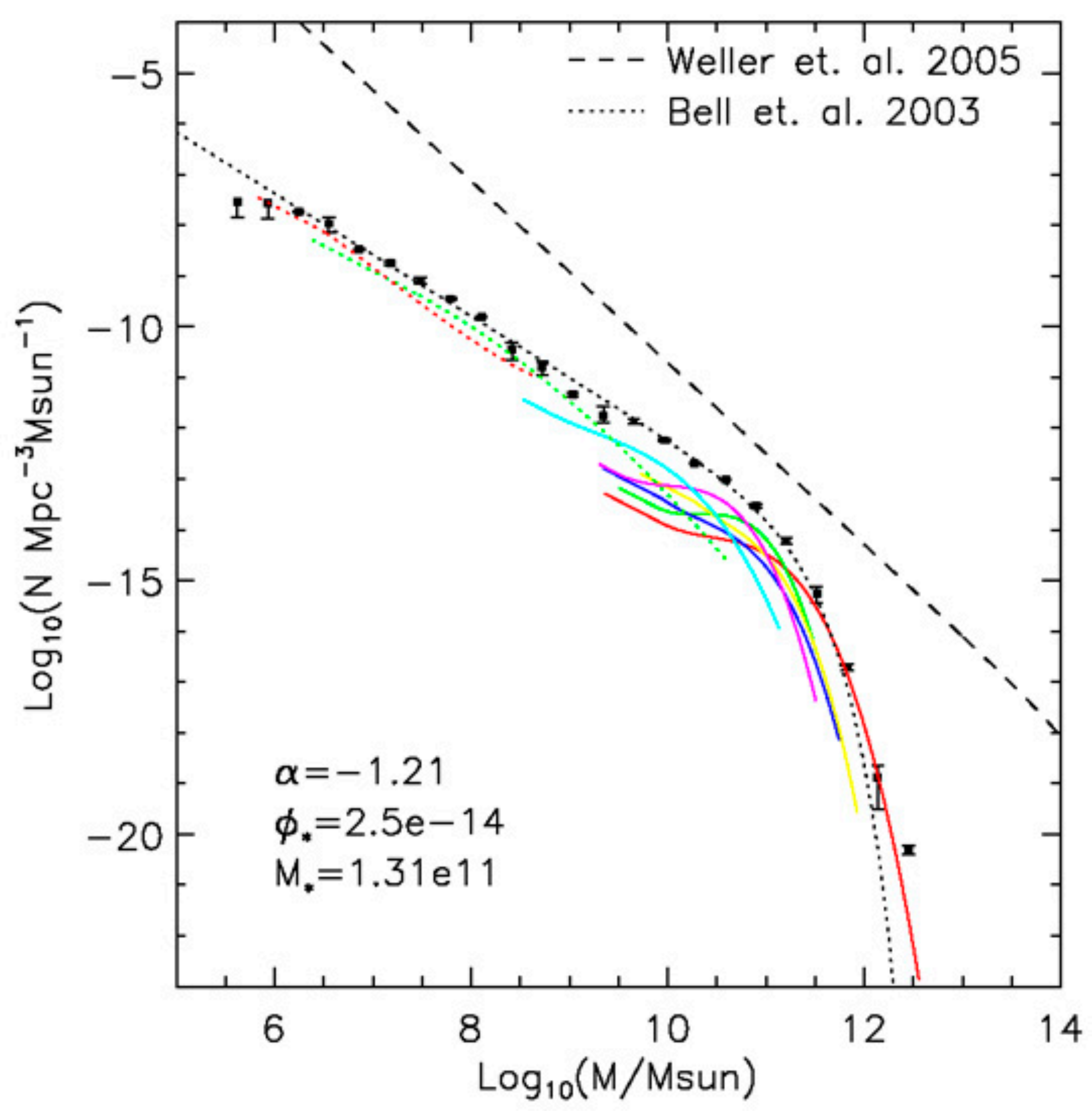


MISSING SATELLITE PROBLEM

- This problem is a little bit overblown, because we already know the galaxy mass function doesn't follow the halo mass function, but it does show how it gets extreme at low mass.

- Many people have suggested changes to dark matter to address this problem, warm dark matter, self-interacting dark matter, or fuzzy dark matter.

- It is also quite likely this is just a consequence of galaxy formation.



EMPIRICAL RELATIONS

Galaxy to Halo Mapping

MAPPING GALAXIES TO DARK MATTER

Now lets turn back to galaxies. It seems like we should be able to get more information from galaxies then just the correlation function and in fact we can.

The next step is to model the connection between galaxies and dark matter, there are a number of ways to do this:

Halo Occupation Distribution: Connect the number of galaxies to the dark matter halo.

Conditional Luminosity Function: Determine a probability of a galaxy of a given luminosity based on the dark matter halo mass.

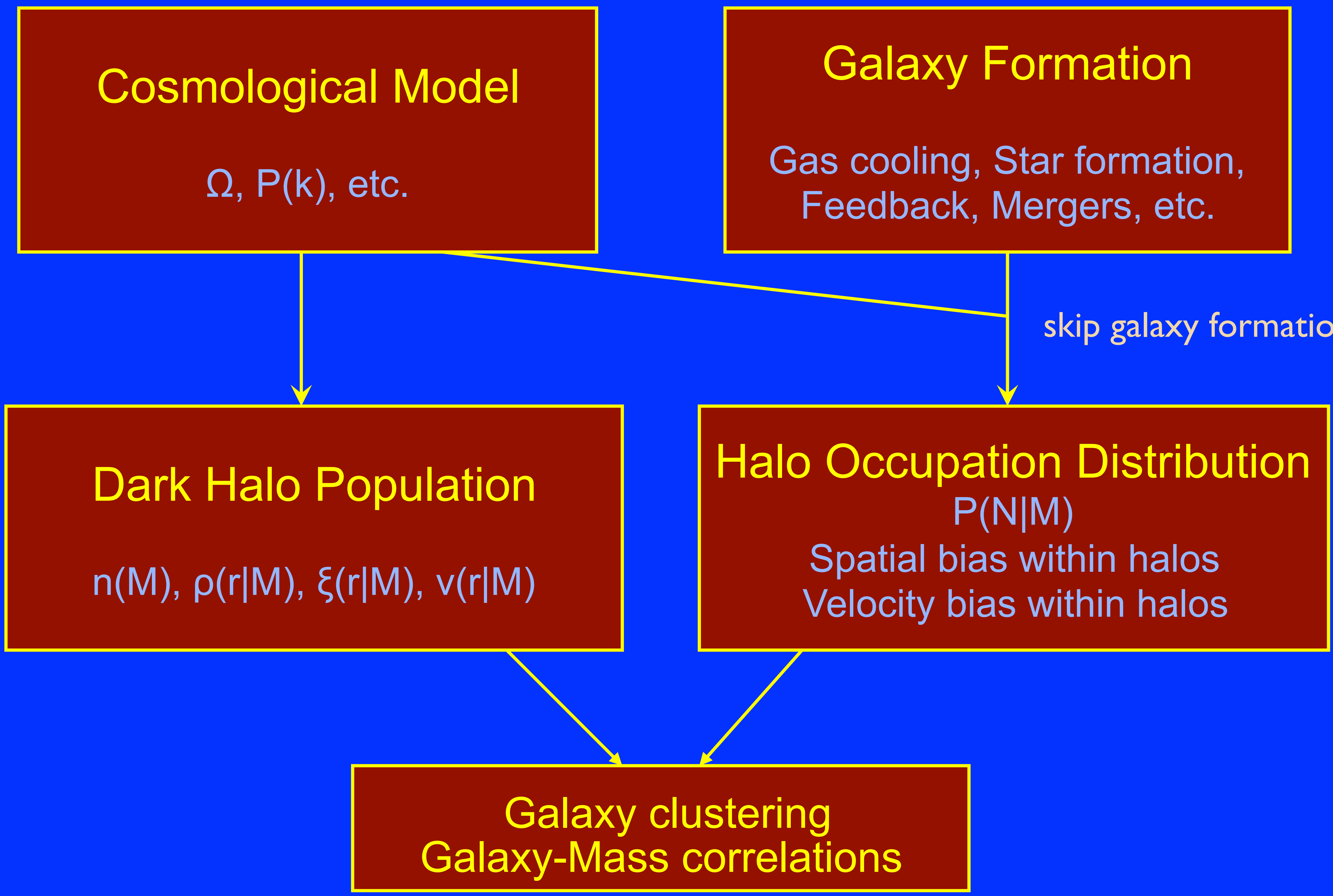
Sub Halo Abundance Matching: Put one galaxy in every sub halo based on some property.

“Halo Occupation” Bias

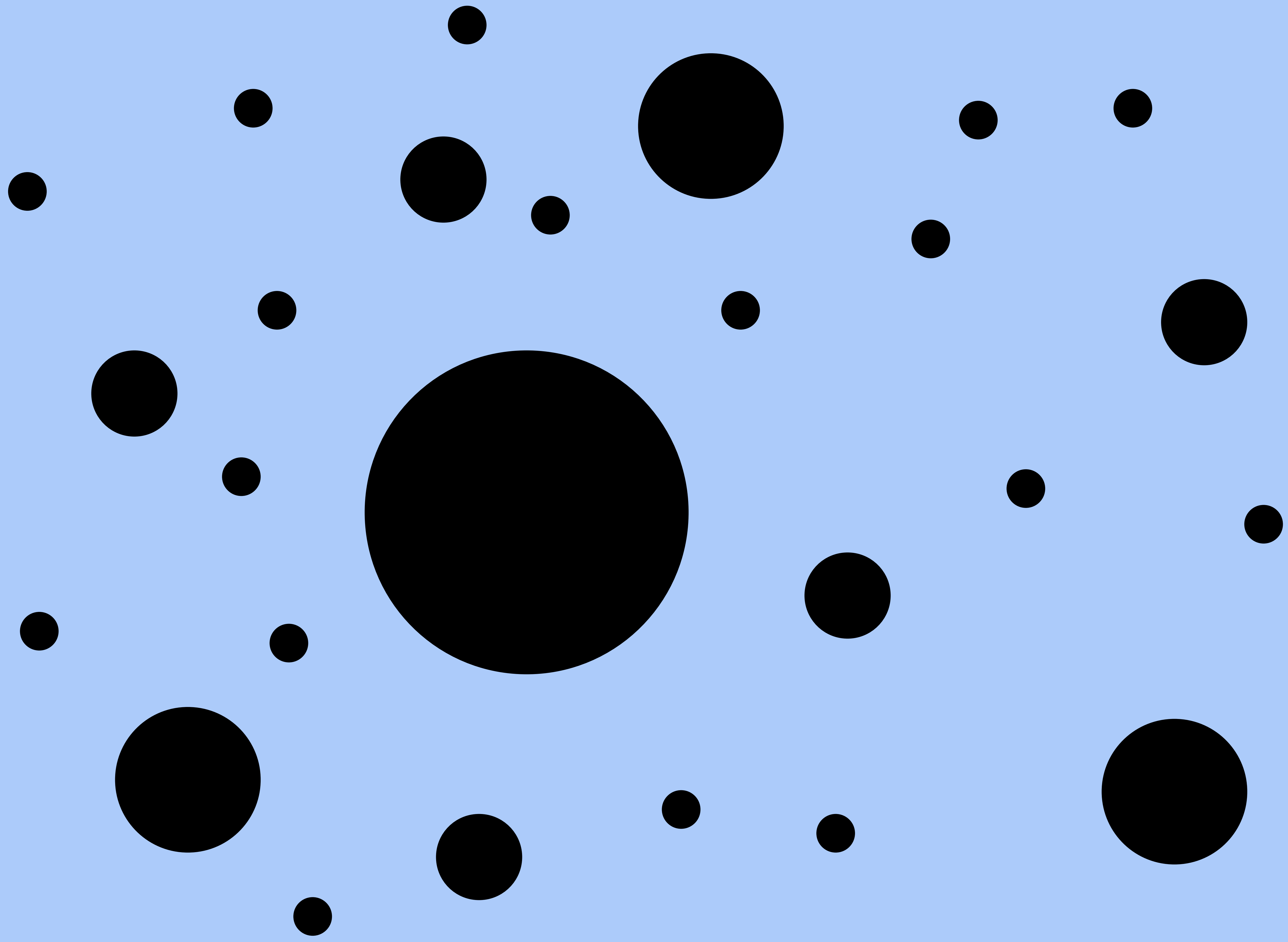
1. All galaxies live in DM halos
2. The galaxy content of a halo is statistically independent of the halo's larger scale environment (depends only on mass)

The bias of any class of galaxies (luminosity, type, etc.) is fully defined by the Halo Occupation Distribution (HOD):

- The probability distribution $P(N|M)$ that a halo of mass M contains N galaxies of that class.
- The relation between the spatial distributions of galaxies and DM within halos.
- The relation between the velocity distributions of galaxies and DM within halos.

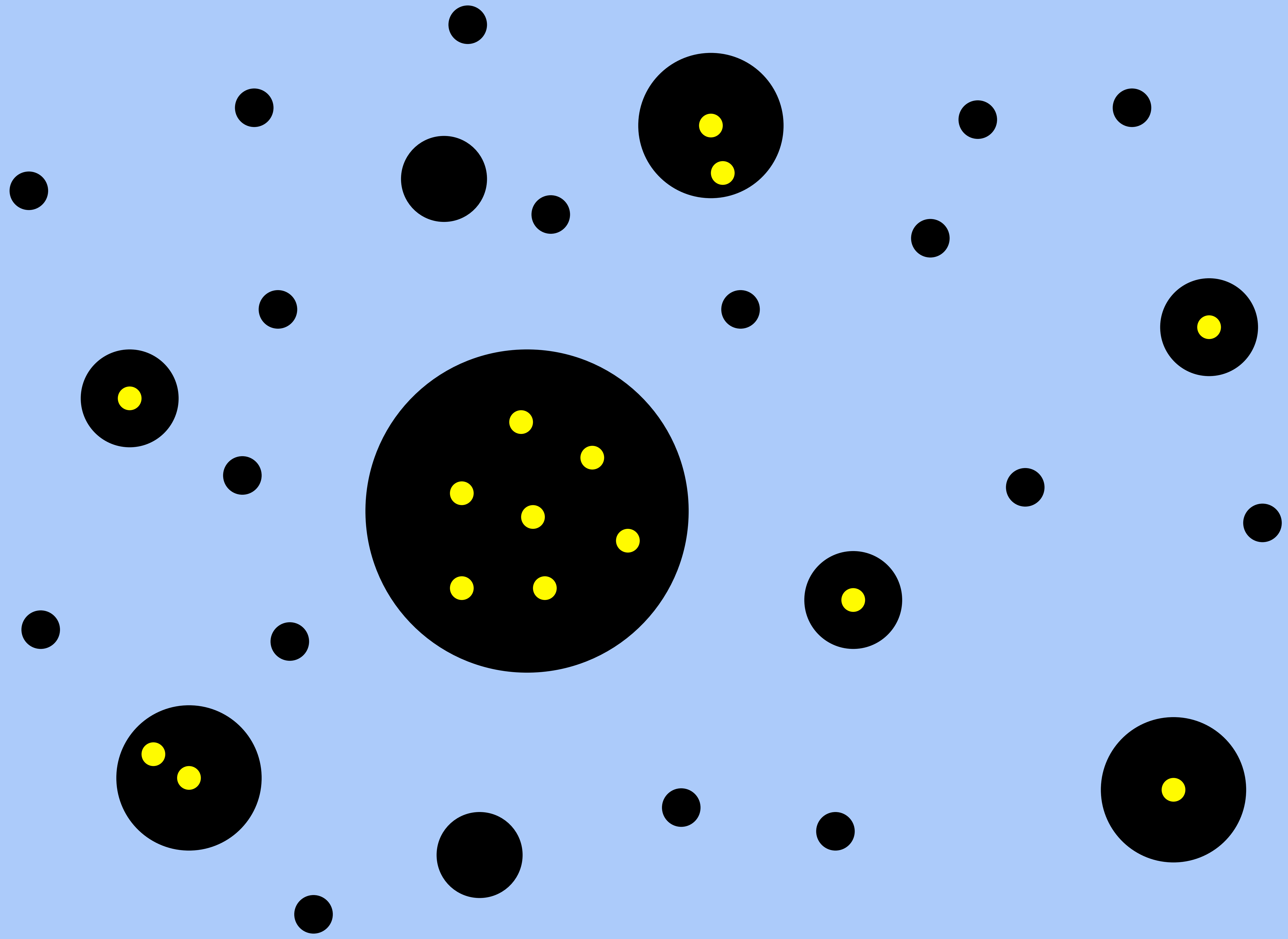

Cosmological Model
Ω, P(k), etc.
Galaxy Formation
Gas cooling, Star formation,
Feedback, Mergers, etc.
skip galaxy formation
Dark Halo Population
n(M), ρ(r|M), ξ(r|M), v(r|M)
Halo Occupation Distribution
P(N|M)
Spatial bias within halos
Velocity bias within halos
Galaxy clustering
Galaxy-Mass correlations

Dark Halo Population



Dark Halo Population

Galaxy Population



Why is the Halo Occupation Distribution (HOD) the right way to think about bias?

- **Complete:** It tells us everything a theory of galaxy formation has to say about galaxy clustering (all statistics, all scales).
- **Physically illuminating:** Discrepancies offer guidance about their physical origin.
- **Observationally powerful:** Description of bias at the level of systems in dynamic equilibrium, where methods can constrain mass.

Nice conceptual division between roles of “cosmological model” and “theory of Galaxy formation”.

The basic approach.

- Develop machinery to compute galaxy clustering statistics given halo properties (mass function, etc.) + HOD.
- We know how to go from cosmological parameters to halo properties.
- Parameterize the HOD (and thus our ignorance about galaxy formation).
- Fit cosmological + HOD parameters (or HOD parameters at fixed cosmology) to galaxy clustering measurements.
- Use measured HODs to gain insight into galaxy formation.

How do we parameterize the HOD?

- Look at theoretical predictions for guidance.
- Interested in moments of *P(N|M)*, as well as radial and velocity distributions within halos.

$$\langle N \rangle_M = \sum_N N P(N|M)$$

$$\langle N(N-1) \rangle_M = \sum_N N(N-1) P(N|M)$$

$$\langle N(N-1)(N-2) \rangle_M = \sum_N N(N-1)(N-2) P(N|M)$$

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How do we parameterize the HOD?

A note about the second moment of integer distributions

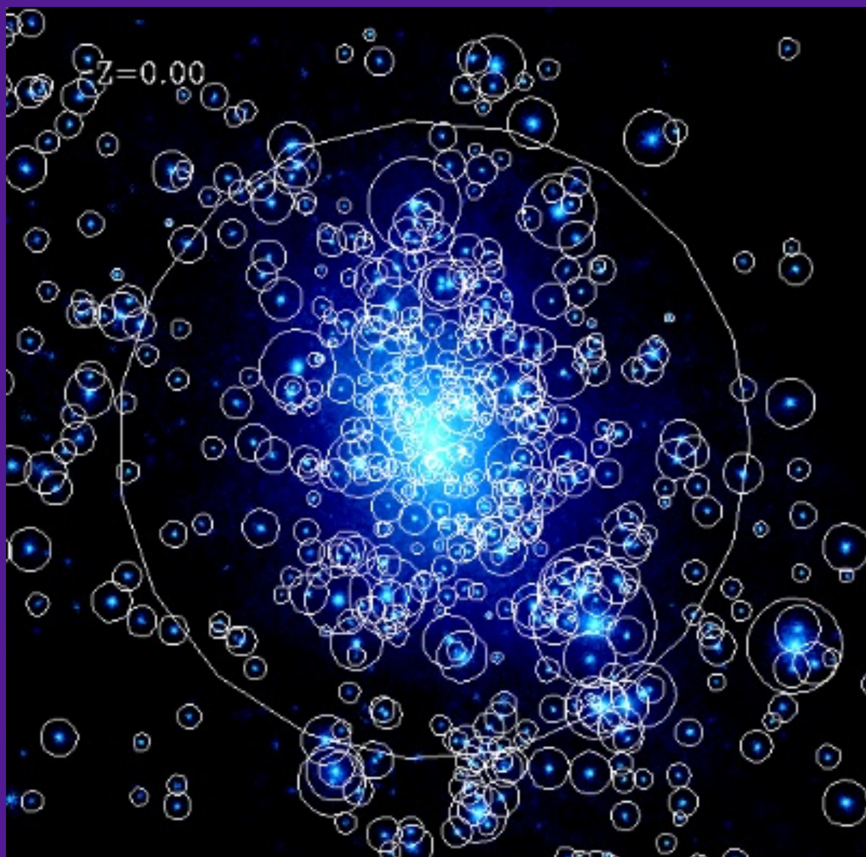
For a Poisson distribution: $$\langle N^2 \rangle = \langle N \rangle^2 + \langle N \rangle$$

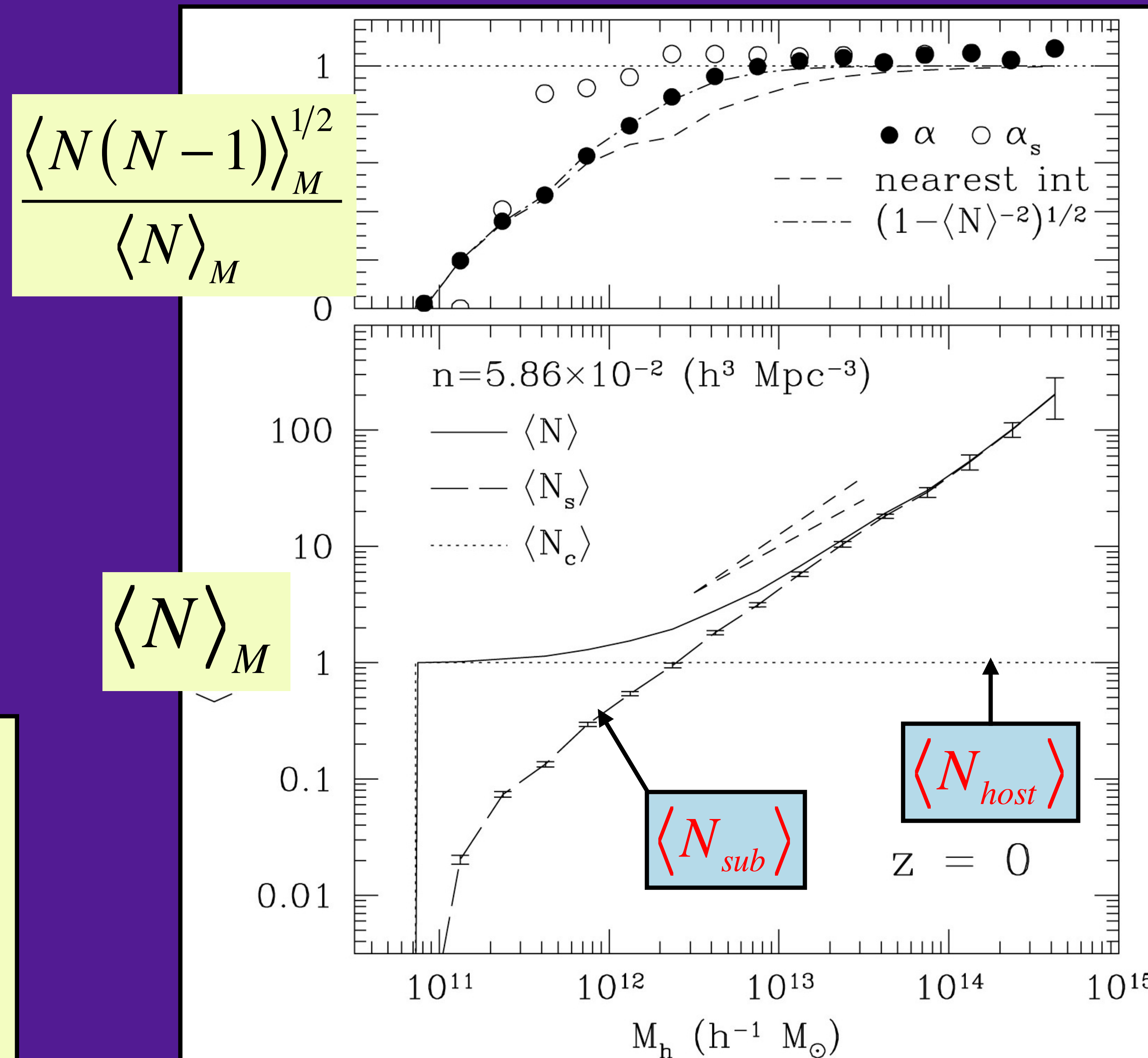
The number of pairs is: $$\langle N(N-1) \rangle = \langle N^2 - N \rangle$$
$$= \langle N^2 \rangle - \langle N \rangle$$
$$= \langle N \rangle^2$$

Narrower distributions have a smaller value than this and wider distributions have a larger value for the number of pairs.

How do we parameterize the HOD?

N-body



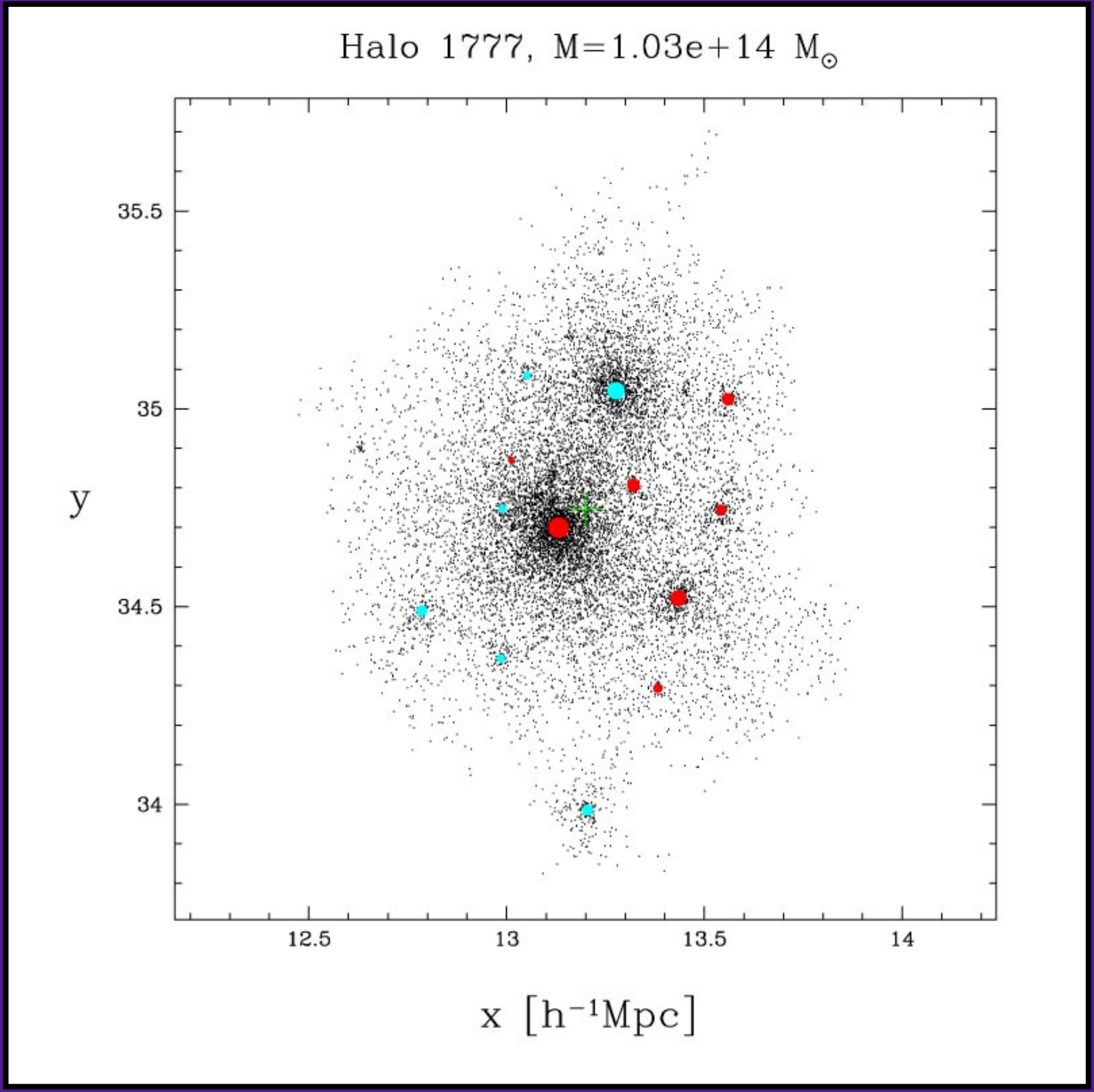


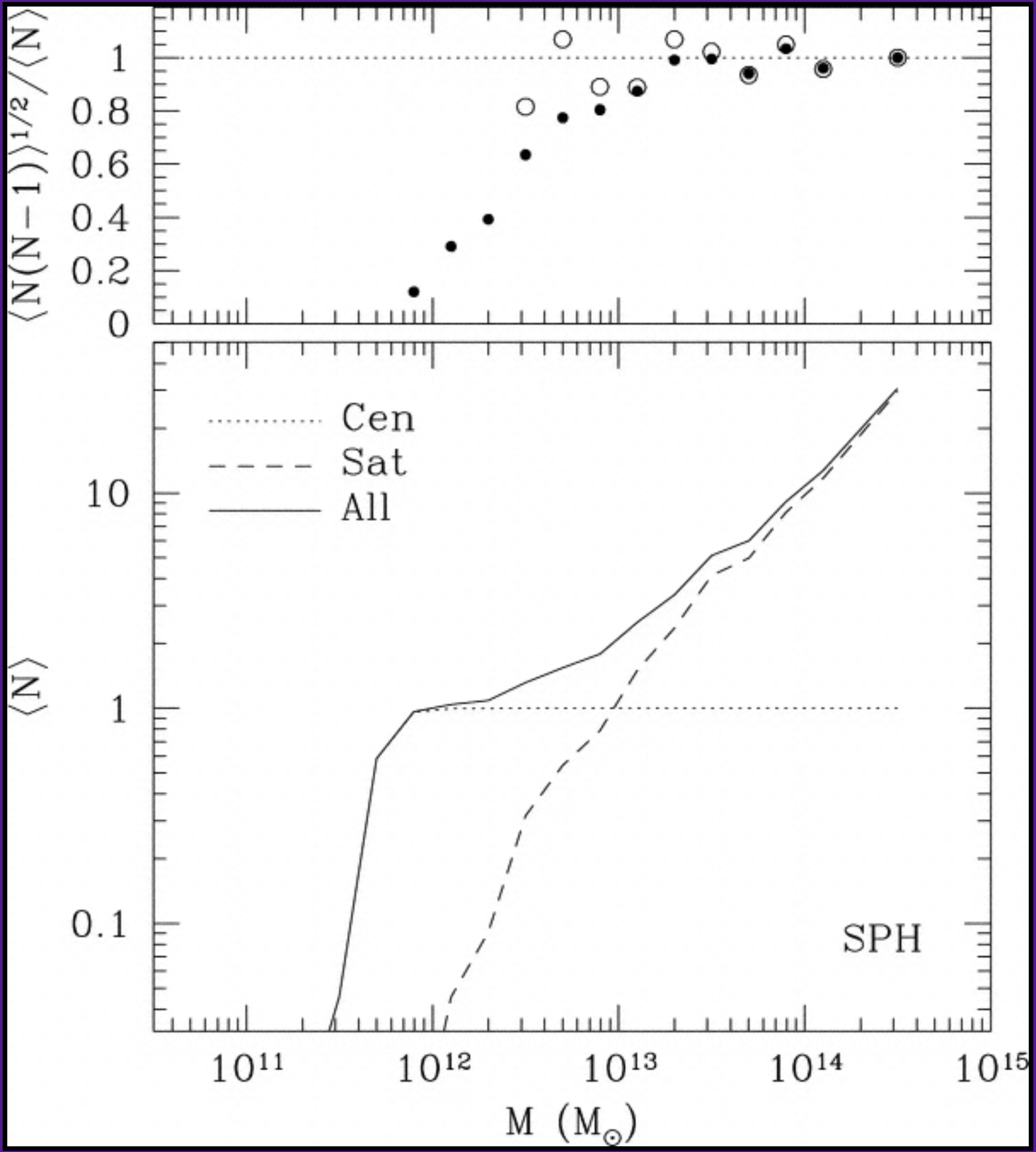
Kravtsov, Berlind et al. (2004)

- HOD for halos + subhalos.
- $\langle N_{sub}\rangle$ is a power law with slope ~1.
- Distribution about $\langle N_{sub}\rangle$ is Poisson.

How do we parameterize the HOD?

SPH





Zheng, Berlind et al. (2005)

How do we parameterize the HOD?

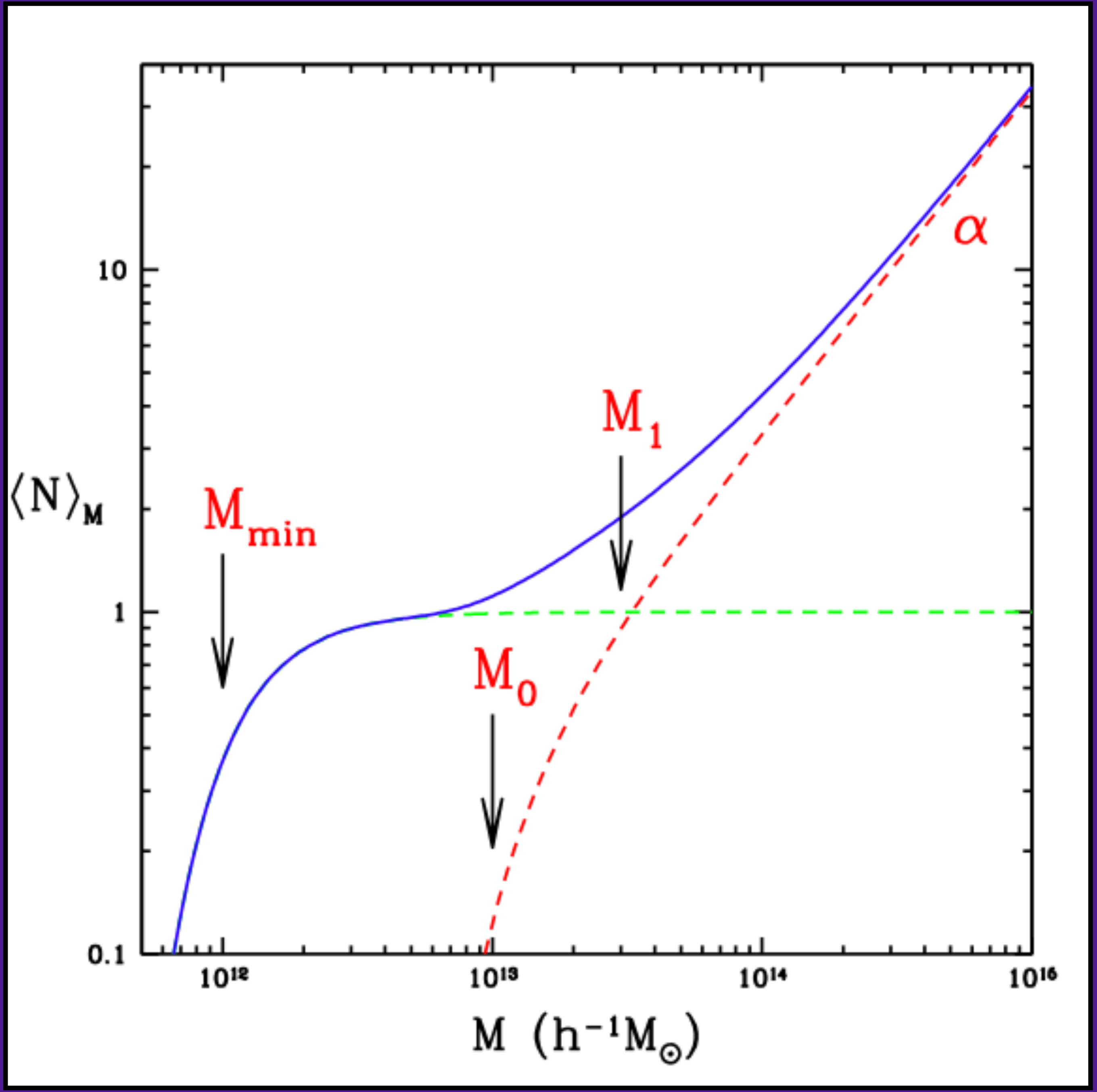
For luminosity/mass threshold samples:

$$N = N_{cen} + N_{sat}$$

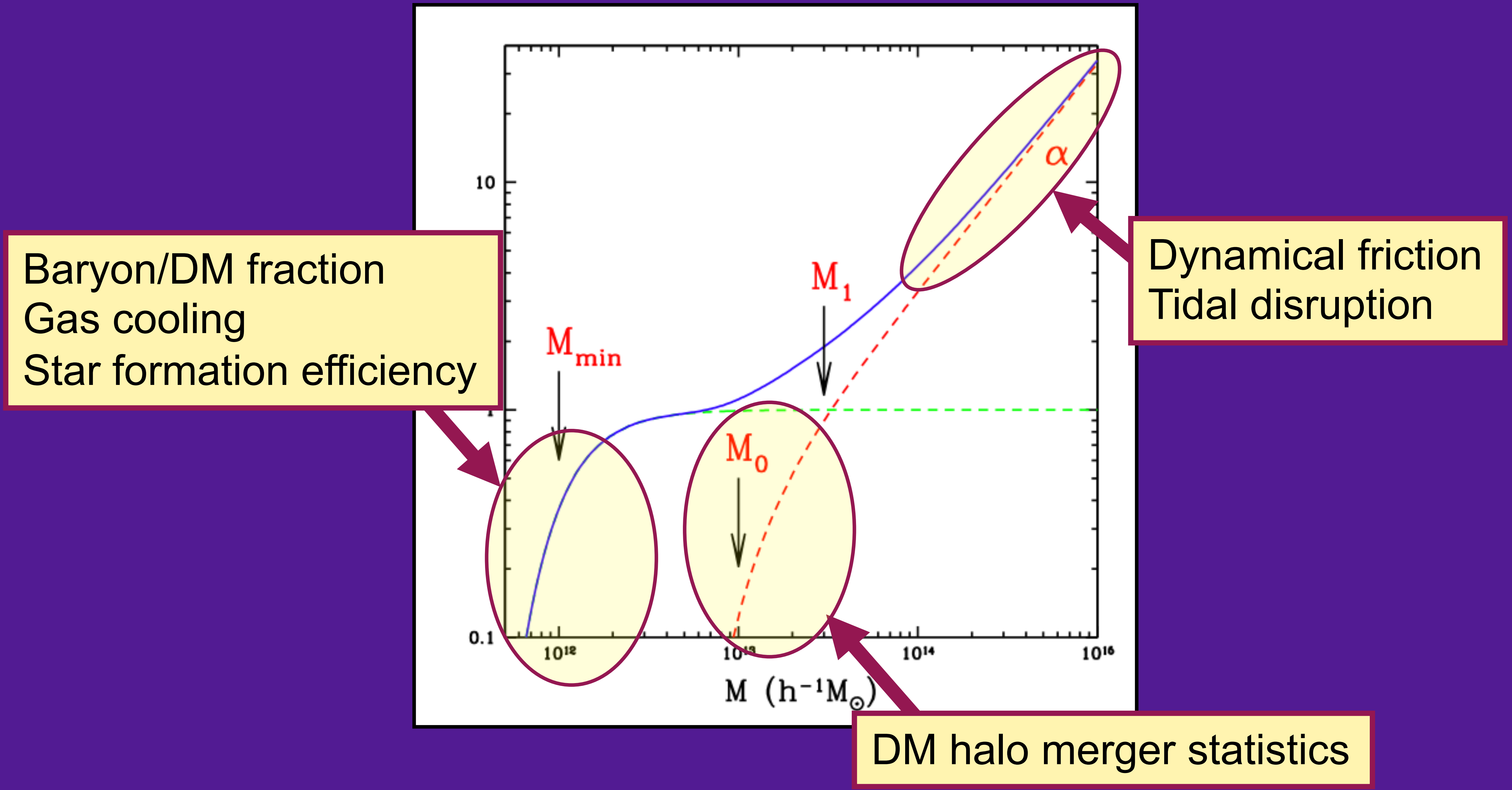
$$N_{cen} = \begin{cases} 0, & M \ll M_{\min} \\ 1, & M \gg M_{\min} \end{cases}$$

$$P\left(N_{sat} \middle| \langle N_{sat} \rangle\right) = \frac{\langle N_{sat} \rangle^{N_{sat}}}{N_{sat}!} e^{-\langle N_{sat} \rangle}$$

$$\langle N_{sat} \rangle = \begin{cases} 0, & M \ll M_0 \\ \left(\dfrac{M}{M_1}\right)^{\alpha}, & M \gg M_0 \end{cases}$$



The HOD contains information about physics!



How do we compute clustering statistics?

Number density

$$n_g = \int_0^\infty dM \, \frac{dn}{dM} \langle N \rangle_M$$

How do we compute clustering statistics?

2-pointCorrelation function

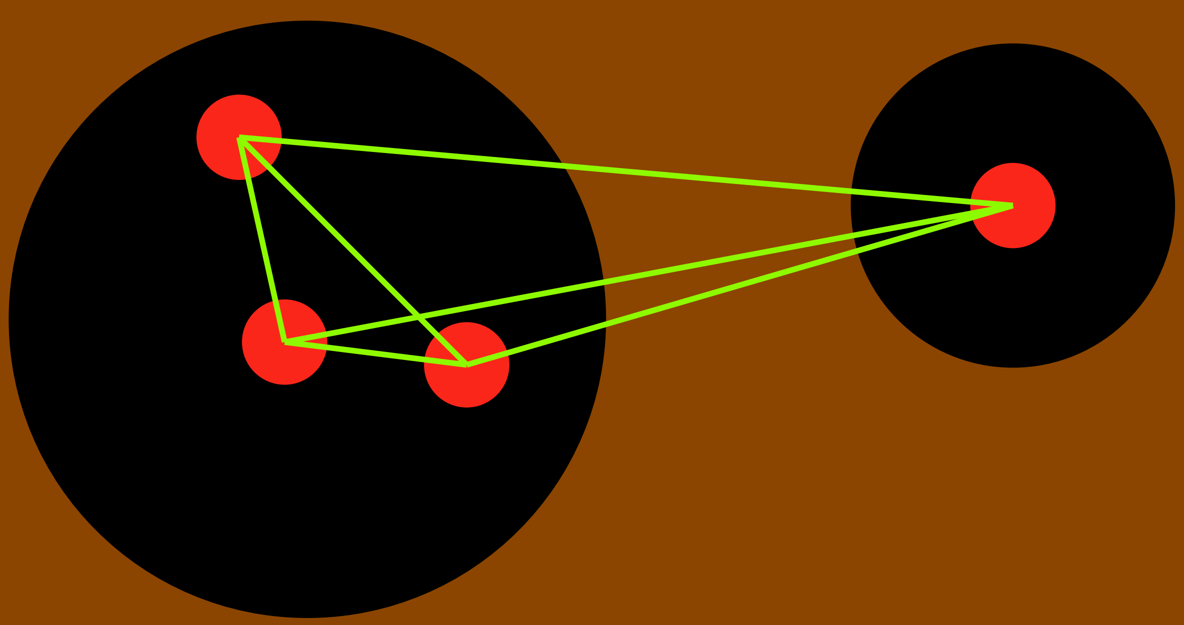
Small scales: All pairs come from same halo.
1-halo term

$$1+\xi_g^{1h}(r)=\left(2\pi r^2 n_g^2\right)^{-1}\int_0^\infty dM\,\frac{dn}{dM}\,\frac{\langle N(N-1)\rangle_M}{2}\,\lambda(r|M)$$

Large scales: Pairs come from separate halos.
2-halo term

$$\xi_g(r)=b_g{}^2\xi_m(r)$$

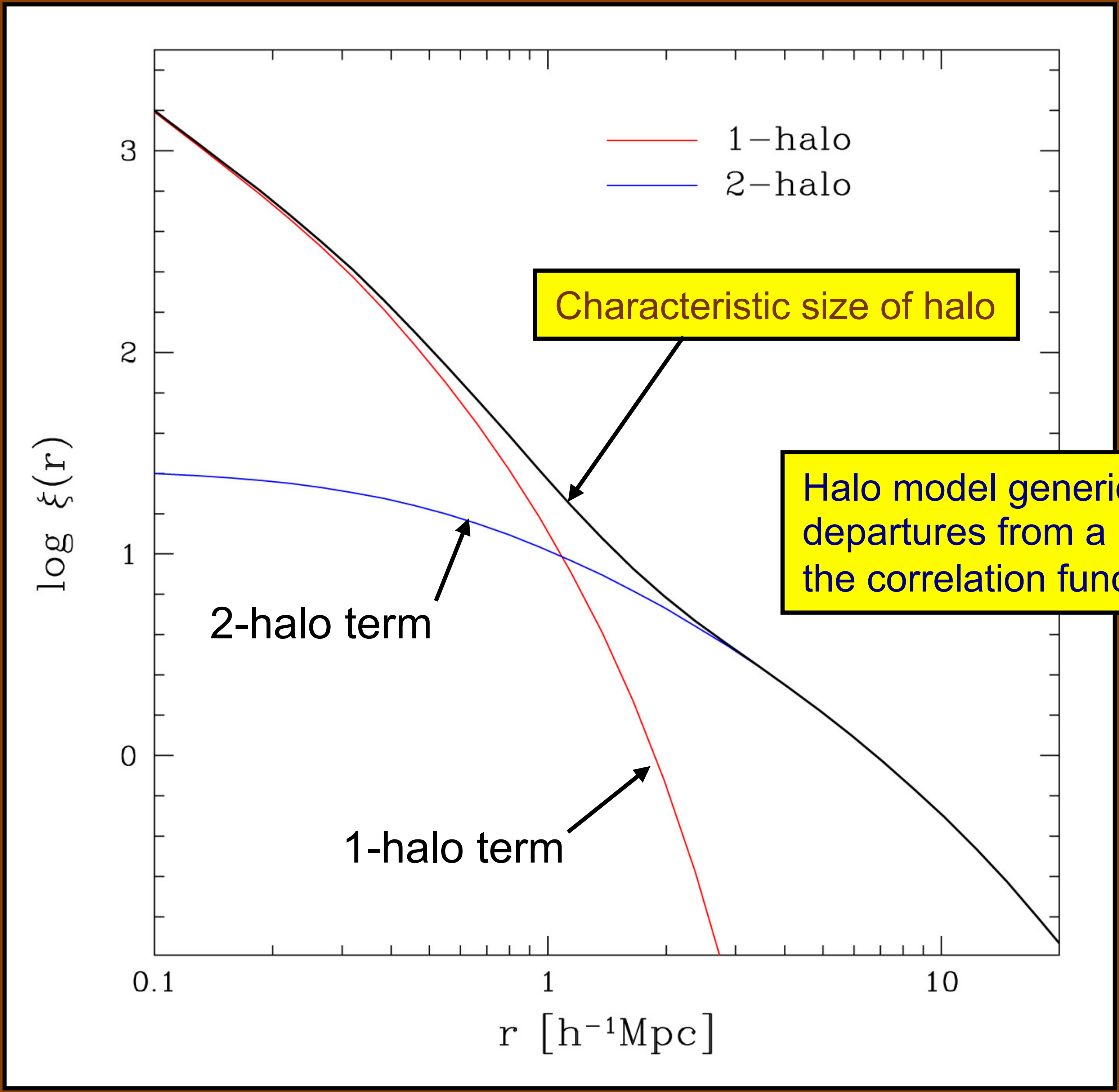
$$b_g=n_g^{-1}\int_0^\infty dM\,\frac{dn}{dM}\langle N\rangle_M b_h(M)$$



Berlind & Weinberg (2002)

How do we compute clustering statistics?

2-point correlation function



Halo model generically predicts departures from a power law in the correlation function.

How do we compute clustering statistics?

N-point correlation functions

3-point function has 3 terms: 1-halo, 2-halo, 3-halo
1-halo term depends on <N(N-1)(N-2)>

Redshift-space and velocity statistics

Need model for velocity distribution in DM halo
+ velocity bias for galaxies

Luminosity function

$$\Phi(L) = \int_0^\infty dM \frac{dn}{dM} \langle N(M,L) \rangle$$

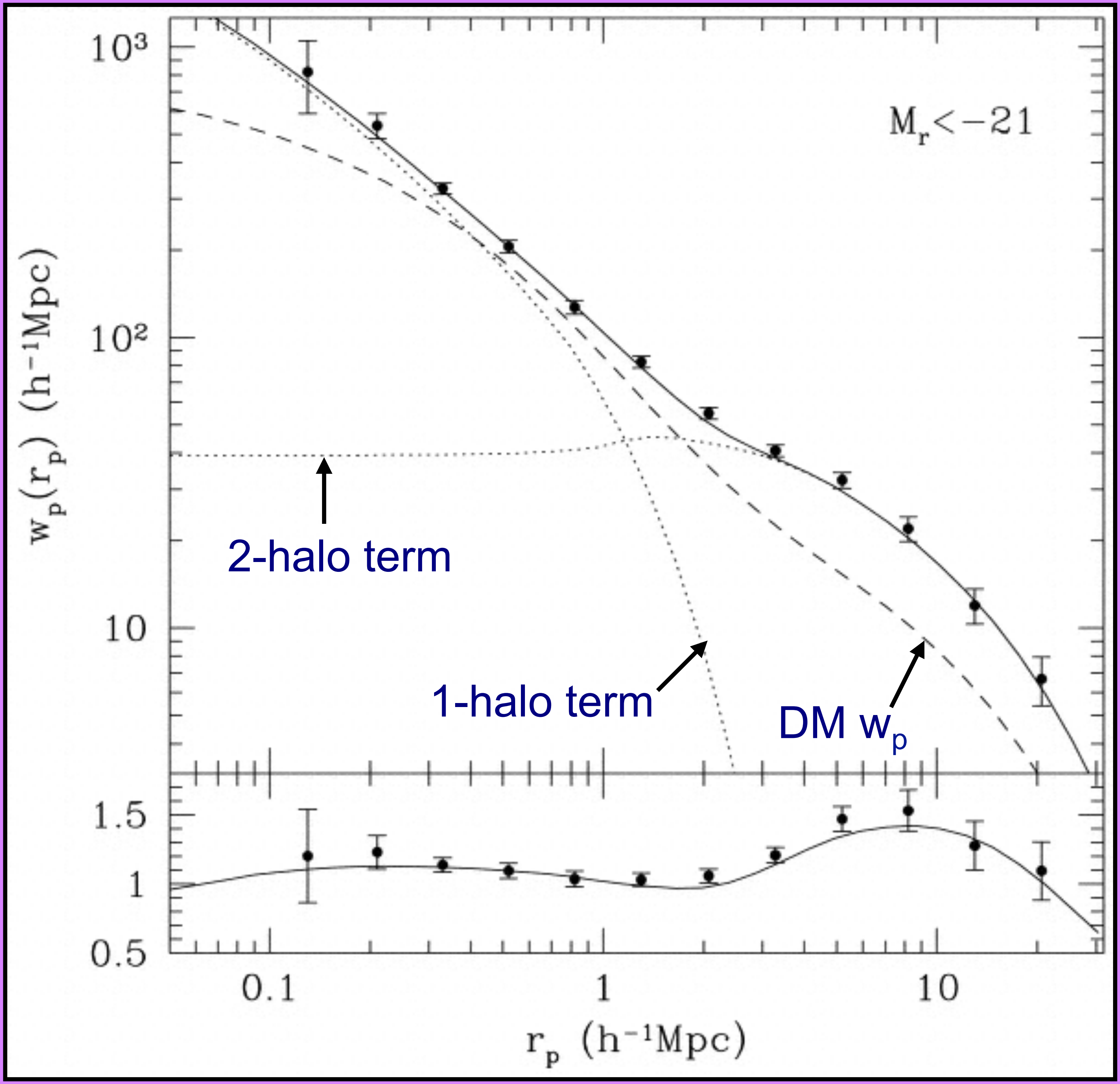
How do we compute clustering statistics?

Improvements to standard halo model

- Non-linear P(k) in 2-halo term
- Scale dependence of halo bias: b(M,r)
- Halo exclusion
- Non-spherical halos
- Non-NFW profiles
- Dependence of b(M) and/or P(N|M) on halo assembly history
- Parameterize P(N|M) for non-trivial galaxy populations

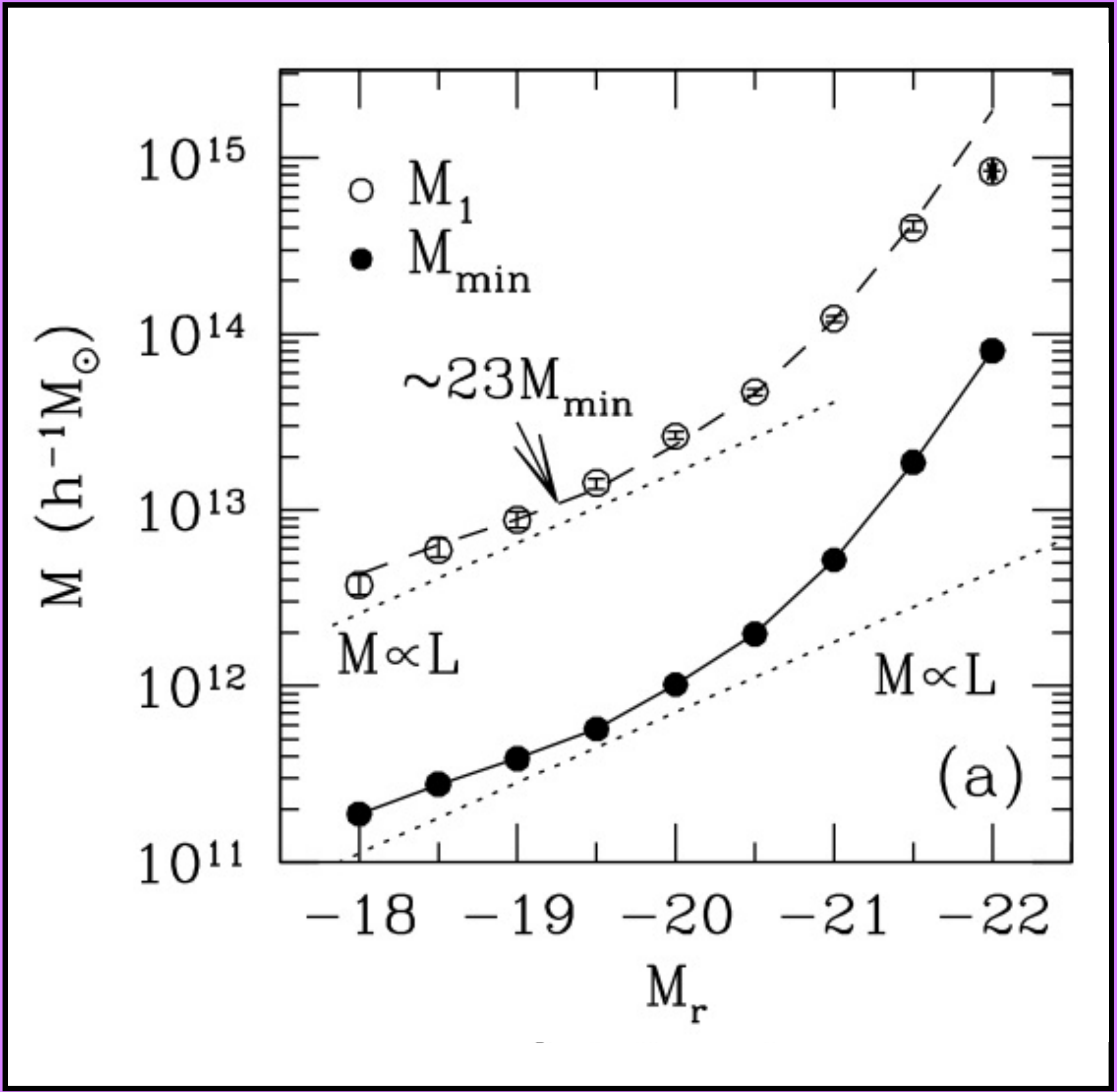
Measurements of the HOD

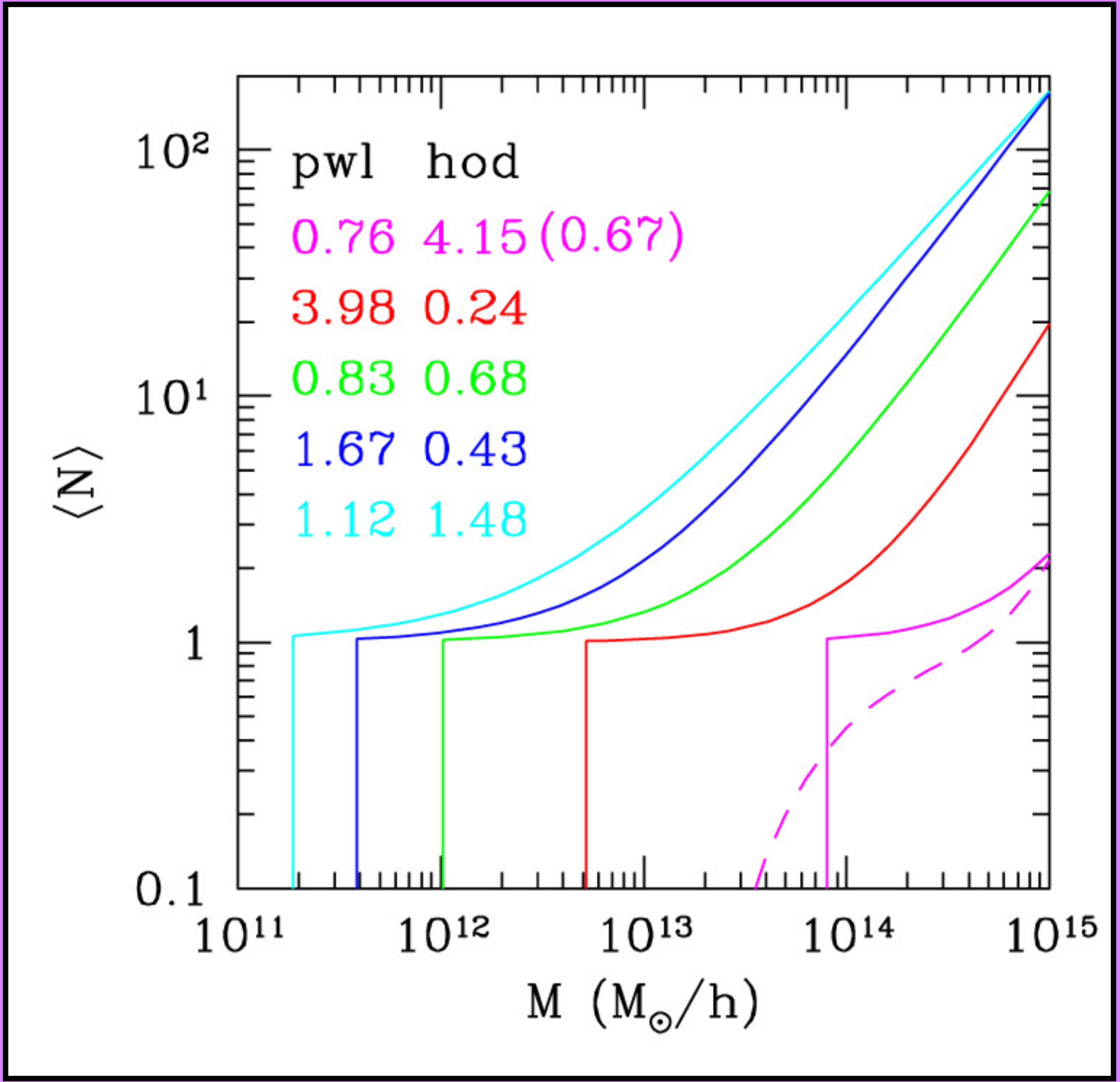
Deviation from power law detected. Halo model gives a good fit to the data.
(χ^2/*dof = 0.93 vs. 6.12 for plaw*)



Zehavi et al. (2004)

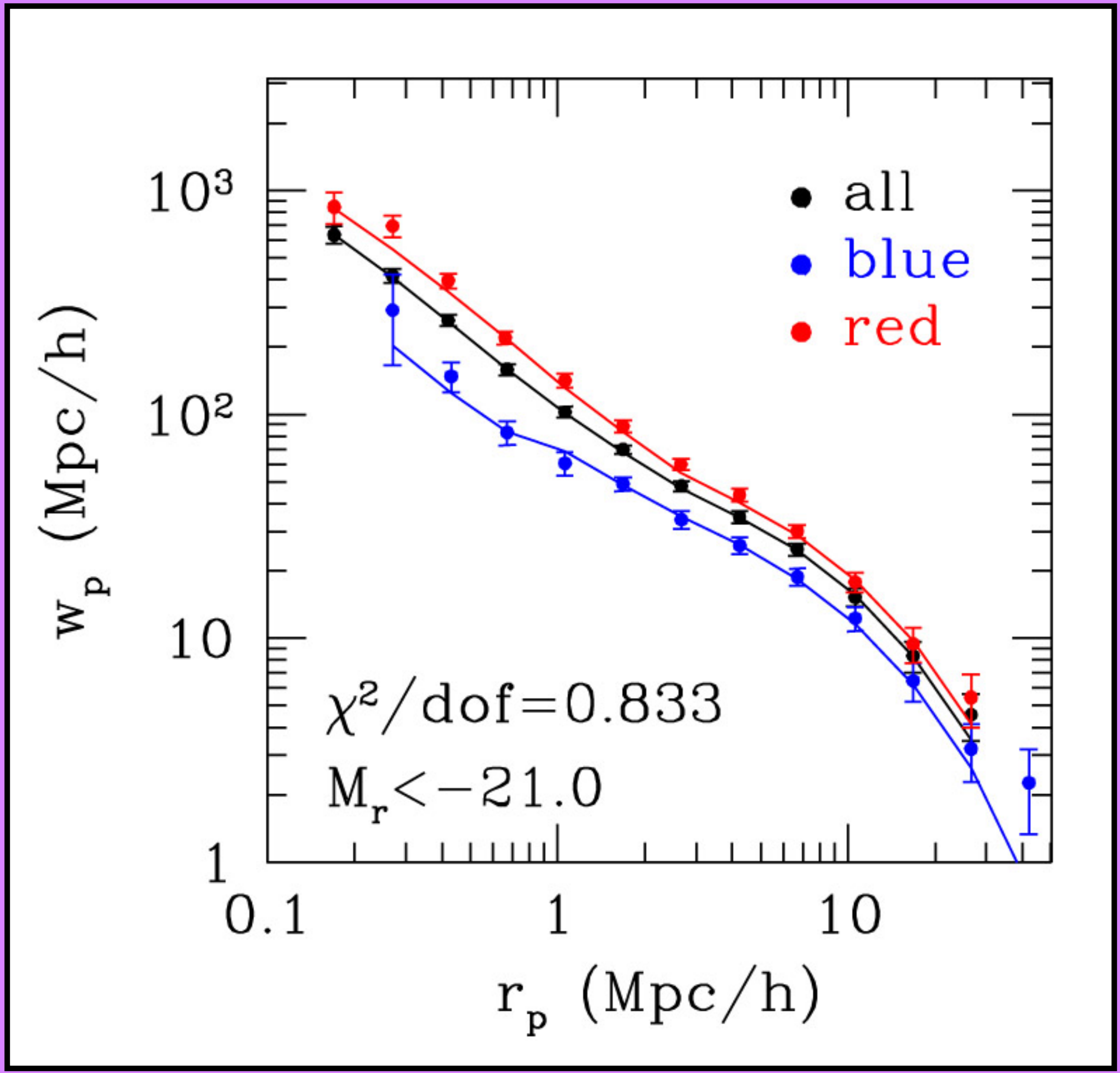
Measurements of the HOD

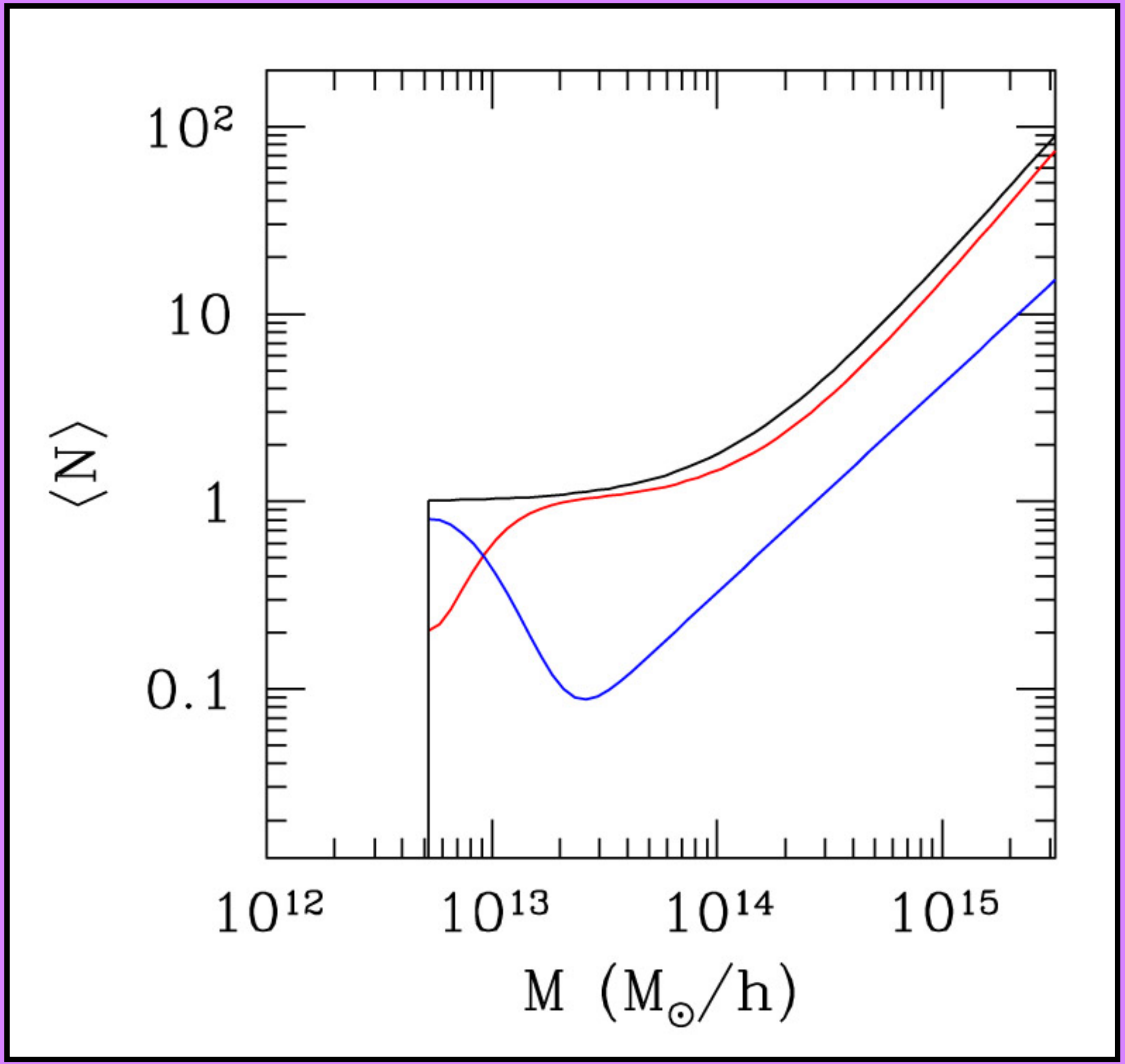




Zehavi et al. (2005)

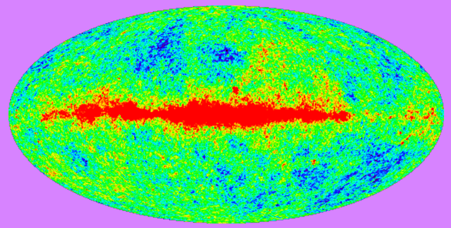
Measurements of the HOD

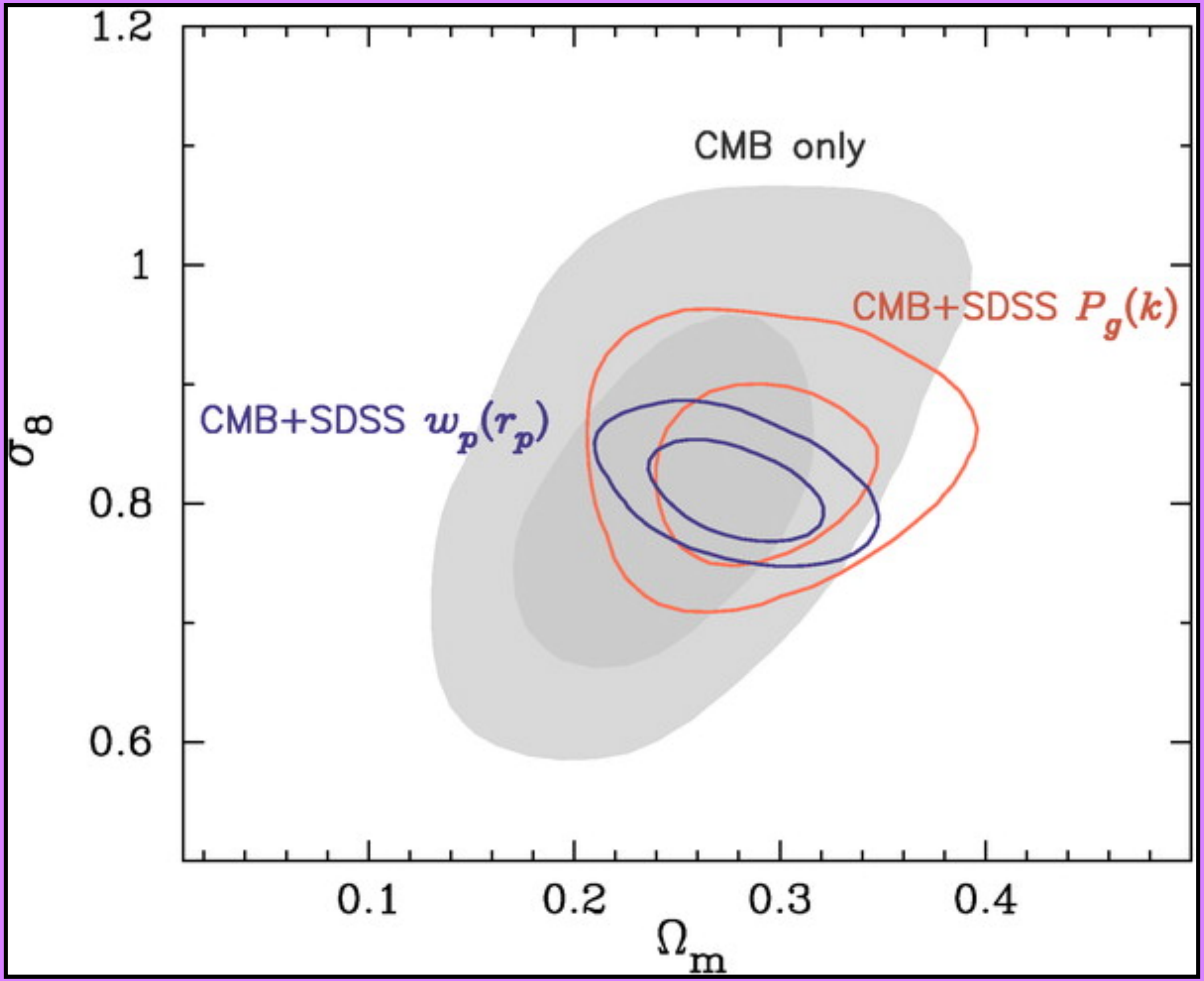




Zehavi et al. (2005)

Measurements of cosmology

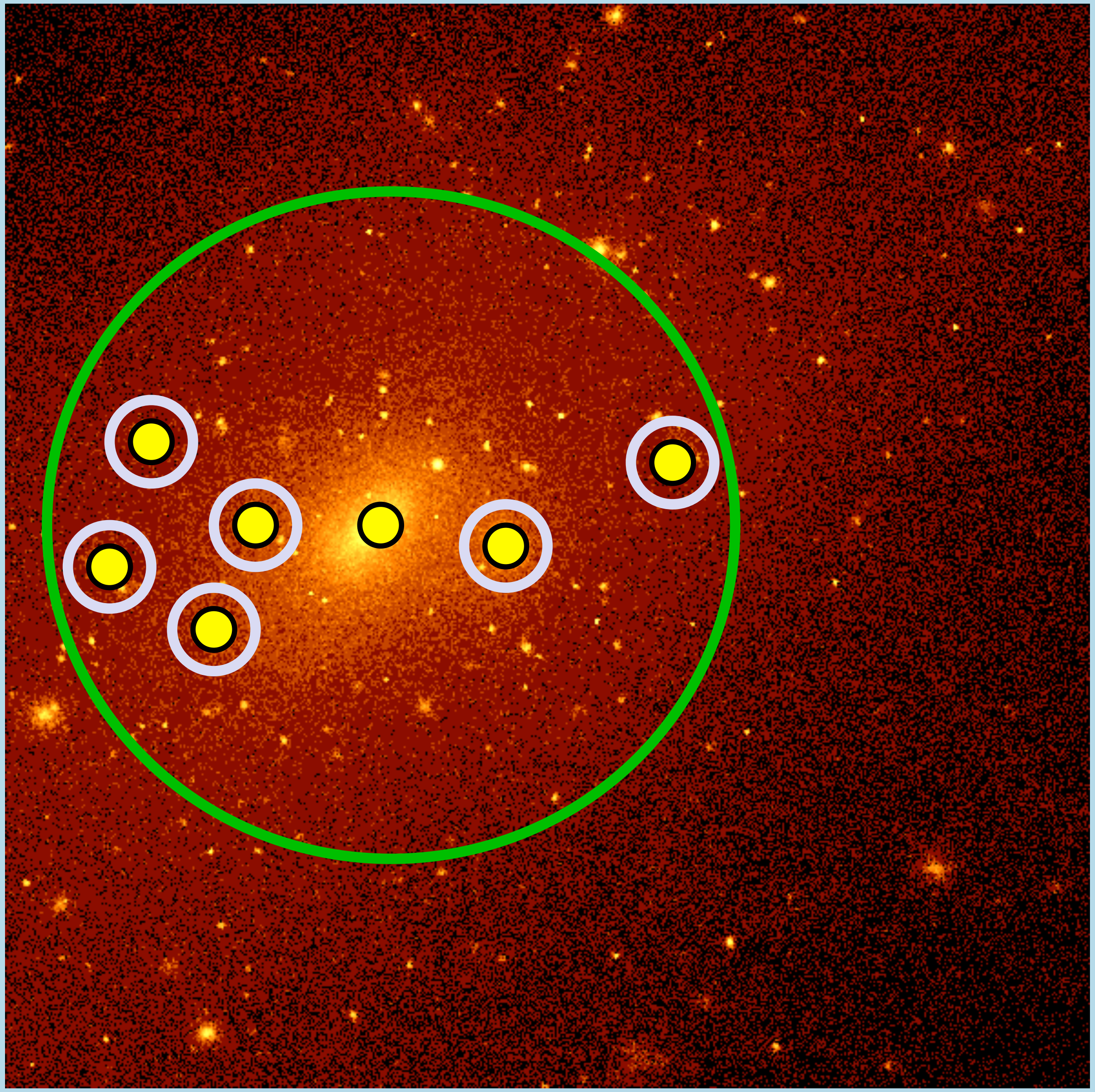




Abazajian et al. (2003)

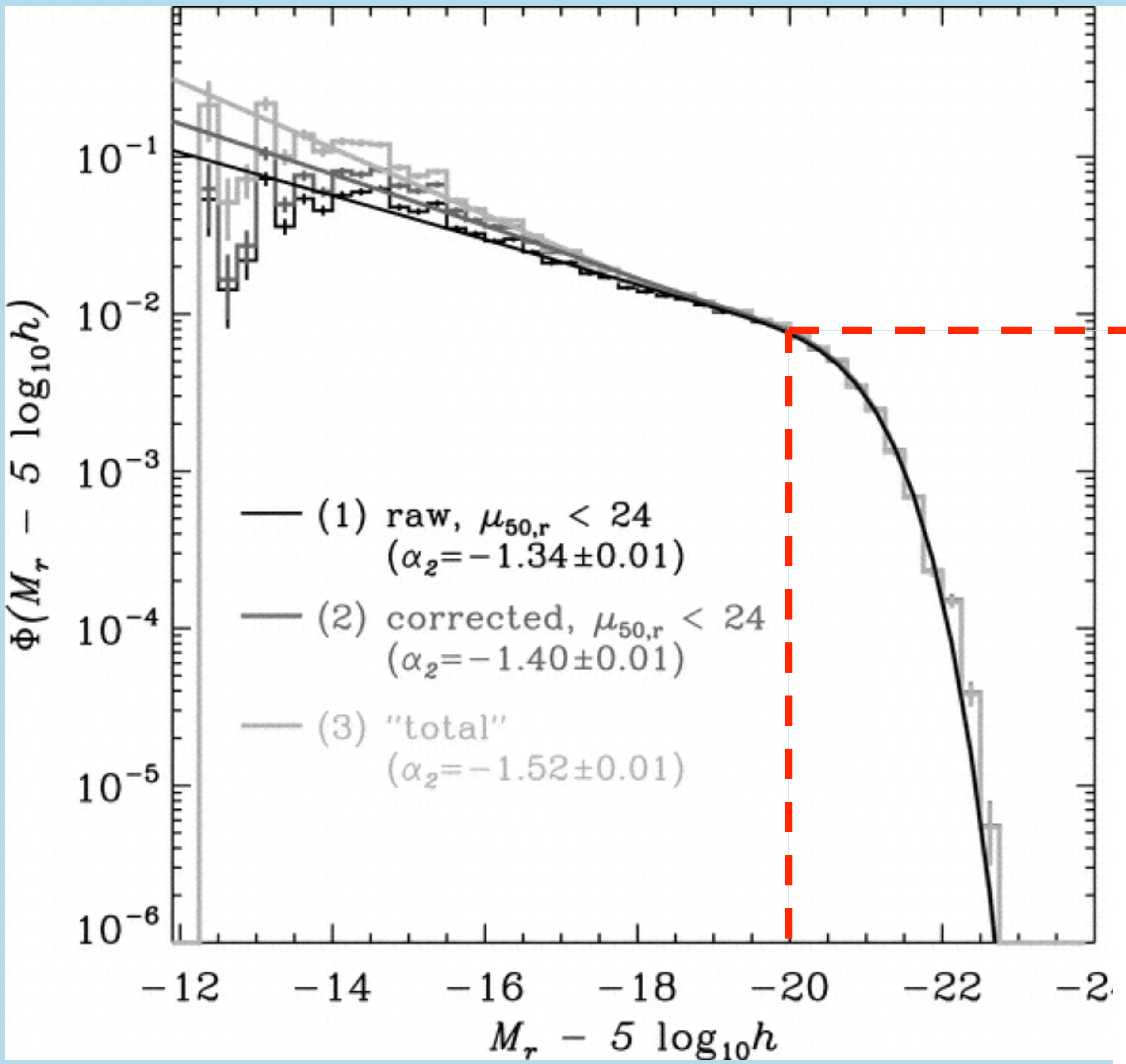
Alternatives to the halo model / HOD approach

Use a high resolution N-body simulation to place galaxies in halos + subhalos, assuming relations between galaxy and subhalo properties. (i.e., use subhalo distribution instead of HOD)



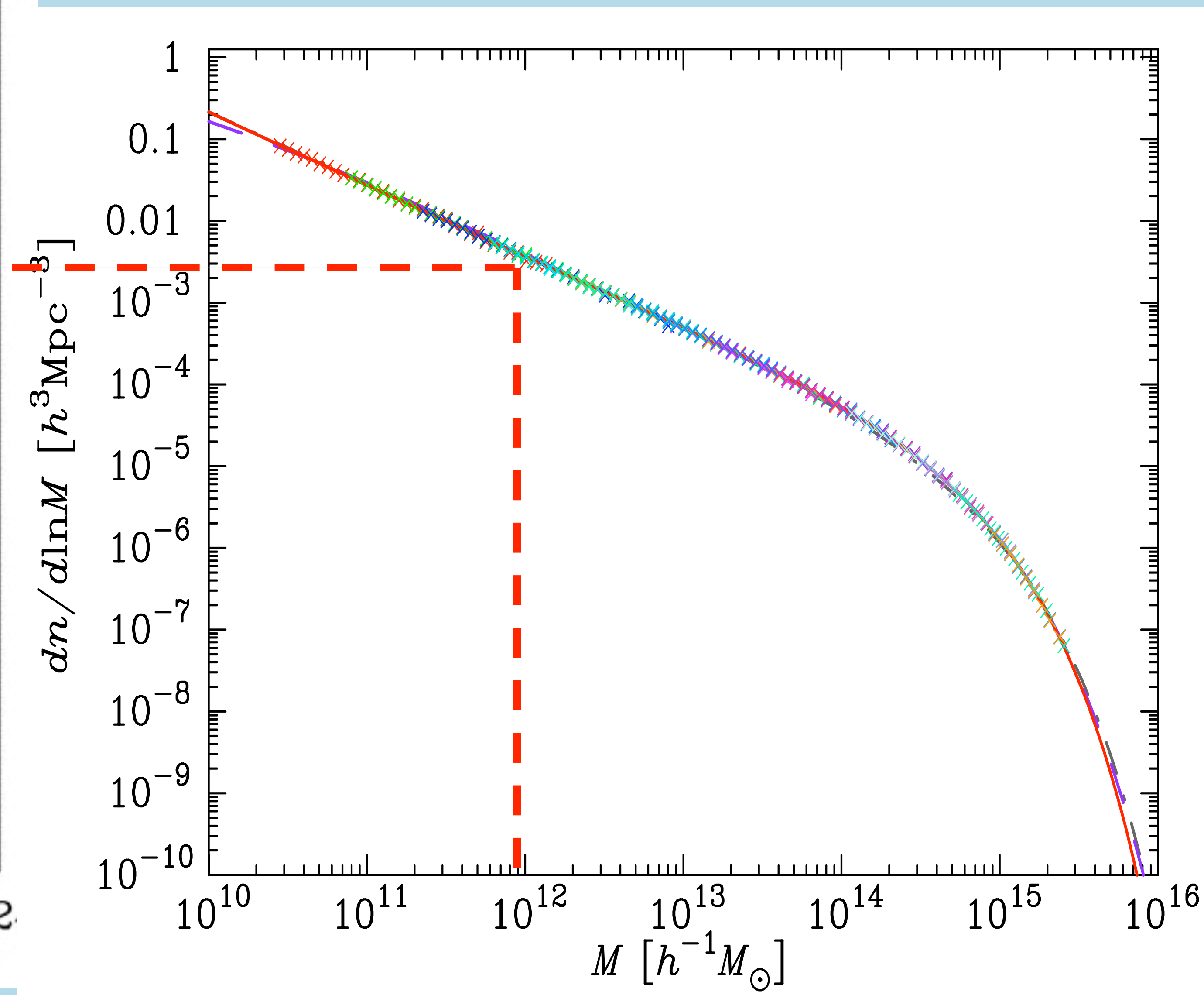
Halo/Subhalo Abundance Matching

Galaxy luminosity function

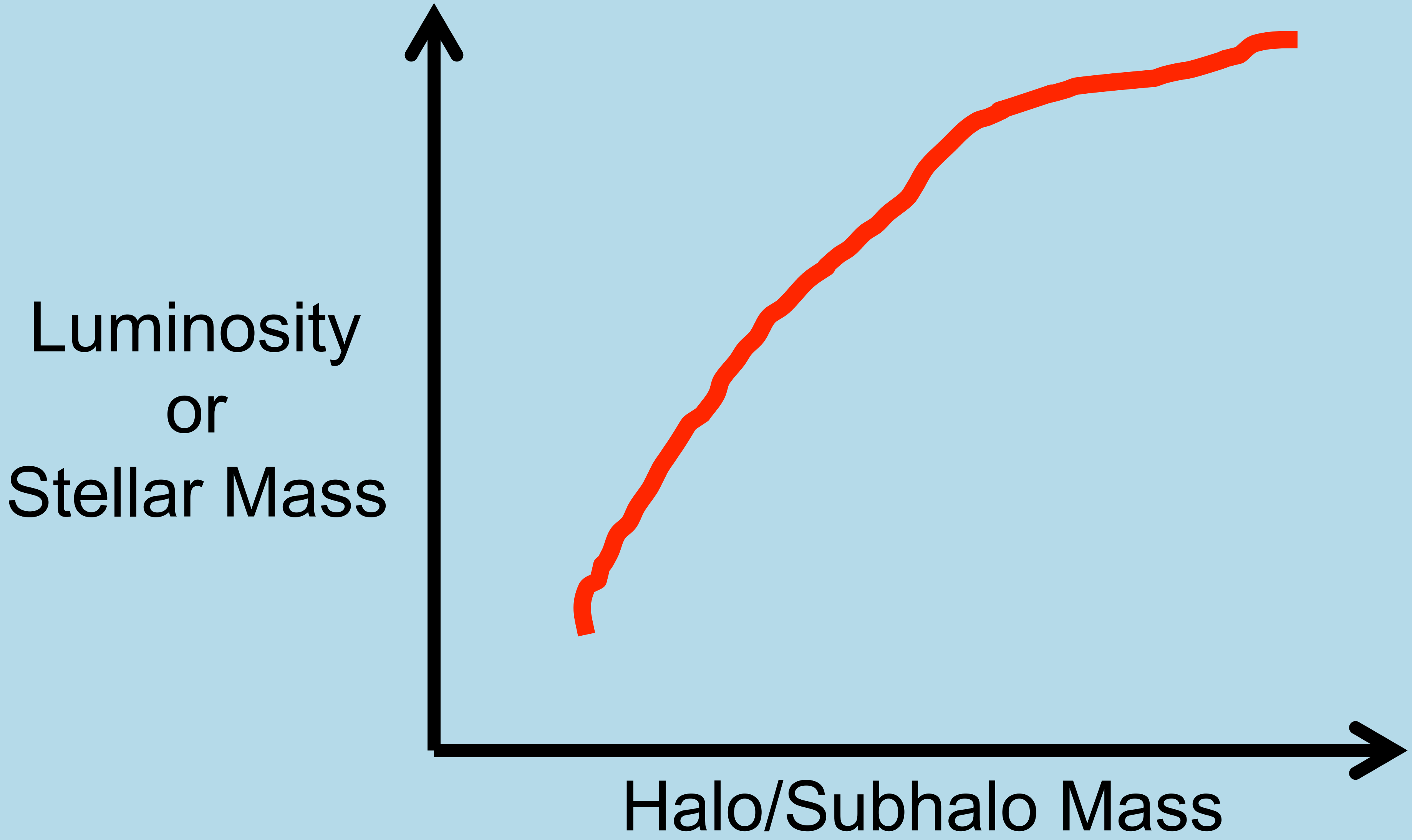


Halo mass function

(including subhalos)

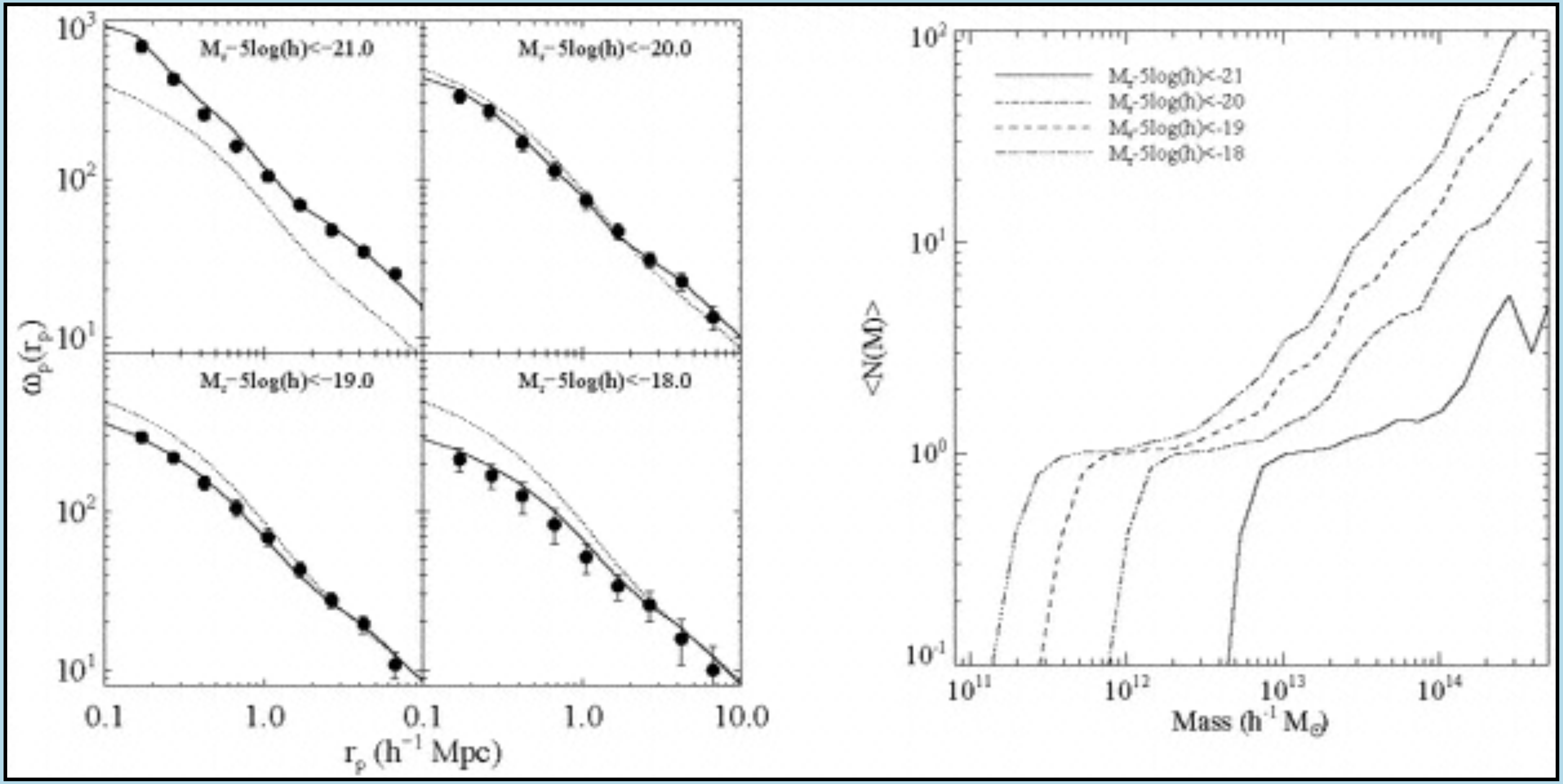


Halo/Subhalo Abundance Matching



Halo/Subhalo Abundance Matching

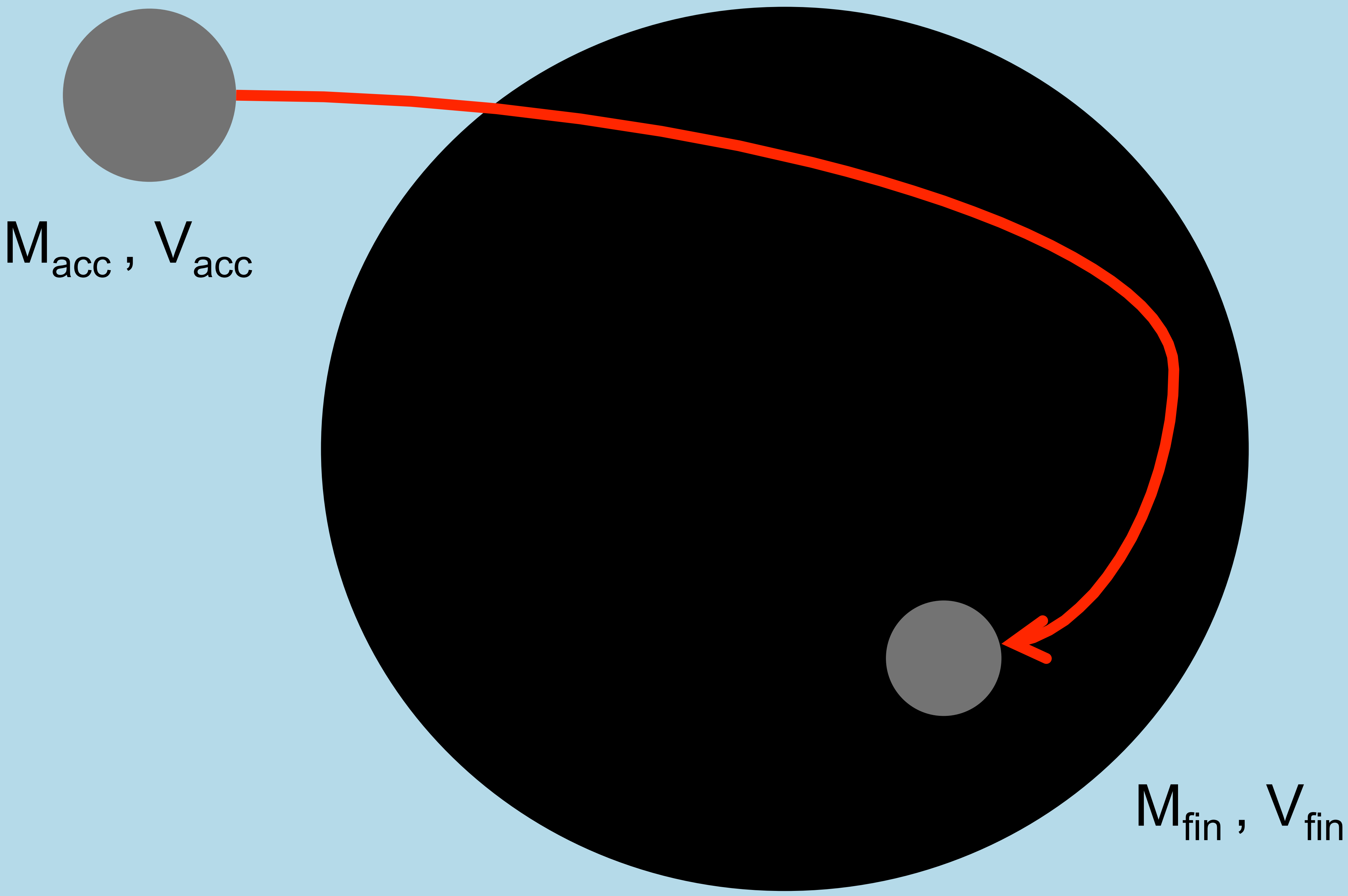
SDSS
z~0



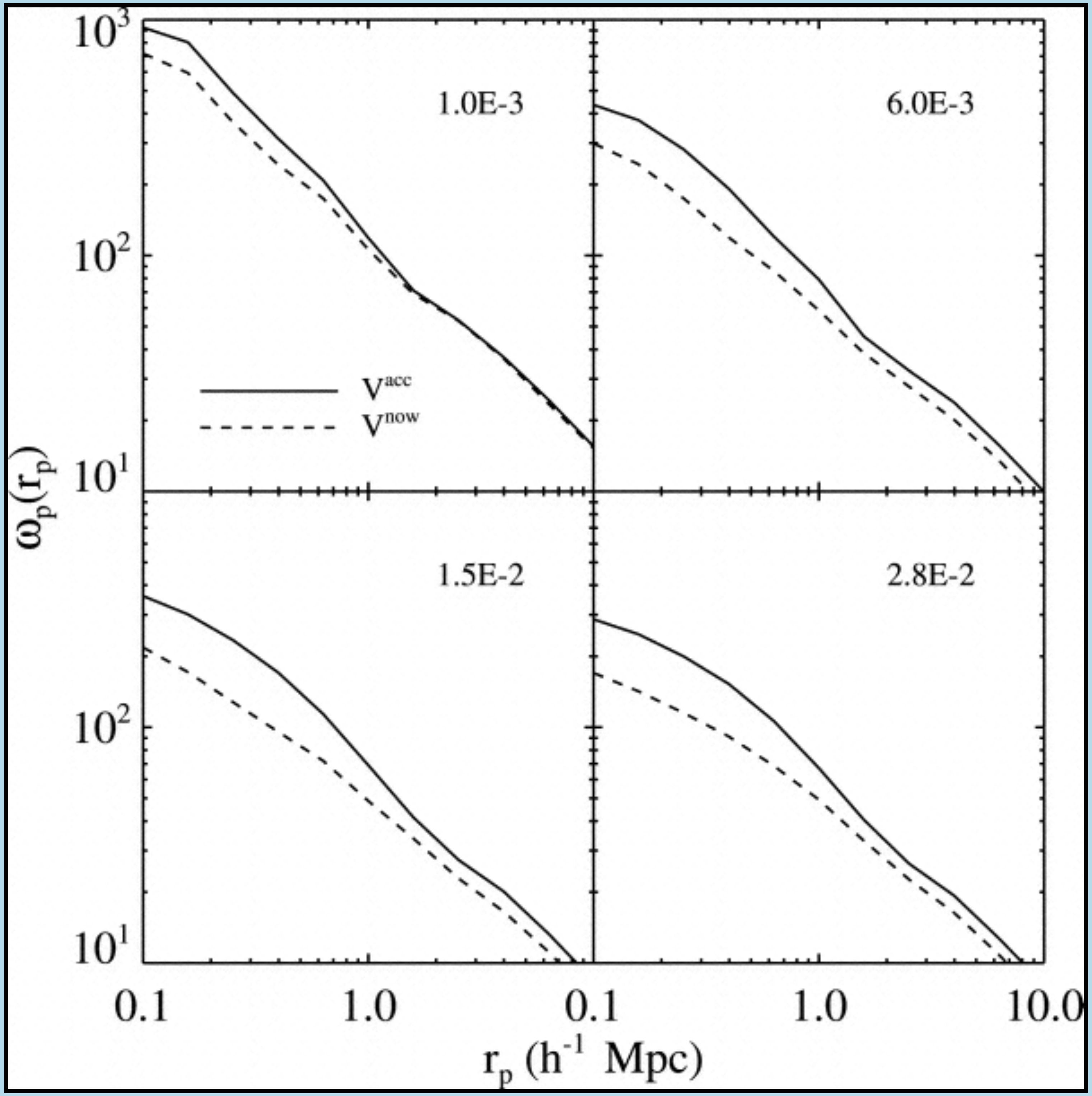
Conroy et al. (2006)

Halo/Subhalo Abundance Matching

What subhalo property should be used?



Halo/Subhalo Abundance Matching



Conroy et al. (2006)

Alternatives to the halo model / HOD approach

Use a high resolution N-body simulation to place galaxies in halos + subhalos, assuming relations between galaxy and subhalo properties. (i.e., use subhalo distribution instead of HOD)

Advantages:

- Let gravity predict what the spatial and velocity distribution of galaxies is.
- Works fairly well for luminosity threshold samples: $L_{gal} \sim M_{sub}$ (Conroy et al. 2006)

Disadvantages:

- Not clear how to populate subhalos with non-trivial galaxy samples (split by color, type, etc).
- Assumes that subhalo evolution within host halos traces that of galaxies.
- Much too slow to constrain cosmology.

Alternatives to the halo model / HOD approach

Use a Conditional Luminosity Function (CLF) to model the luminosity dependence of clustering.

$$\Phi(L) = \int_0^\infty dM \frac{dn}{dm} \Phi(L|M) \qquad \langle N \rangle_M = \int_{L_{\min}}^\infty dL \Phi(L|M)$$

Advantages:

- Don't have to assume a form for <N(M)>
- More ambitious: model the luminosity dependence explicitly

Disadvantages:

- Have to assume a form for $\phi(L|M)$
- More ambitious: luminosity dependence is model dependent

Methods are very similar and complementary.