COSMOLOGICAL PARAMETERS

OUTLINE

* Standard Candles and Rulers
* Galaxy Correlation Functions
* The Cosmic Microwave Background
* Gravitational Lensing
*

SO FAR

We have seen so far that our cosmological model has a number of parameters. They FLRW-model has three parameters, H_0, Ω_m and Ω_Λ. From these three the age of the Universe t_0 and the curvature $\Omega_K = 1 - \Omega_m - \Omega_\Lambda$ can be derived.

However, this does not include any fluctuations. The simplest parameterization of the power spectrum is a power law with index n_s and normalization σ_8.

The most basic model that fits all observations is called the ΛCDM model. It is a 6 parameter model, H_0, Ω_b Ω_{cdm}, Ω_Λ, n_s and σ_8.

MAYBE MORE

But there are many more parameters that can be added to the model. Some we know must be there, but simply have a small effect. Some are additions to the model that may or may not be there.

Matter Fields: We know besides dark matter and baryons there are neutrinos and photons, Ω_ν and Ω_γ, the neutrino number density we know so cosmology places and upper limit on the sum of the neutrino masses. The photon density hasn't been important for most of the history of the Universe but it was at the time of BBN which depends on the photon-to-baryon ration, η_γ.

Dark Energy: Has no free parameters if we assume w=-1, but we can loosen that. We can consider the value today w_0 and how it change with z (or a or t) denoted by w' or w_1.

CURRENT STATUS

Measuring cosmological parameters used to be about the big picture, does Ω_m equal 0.2 or 1? is the Universe flat? is their a cosmological constant? Does CDM give the right power spectrum?

Now days these are measured to a few percent accuracy. The question now has become is there anything more?

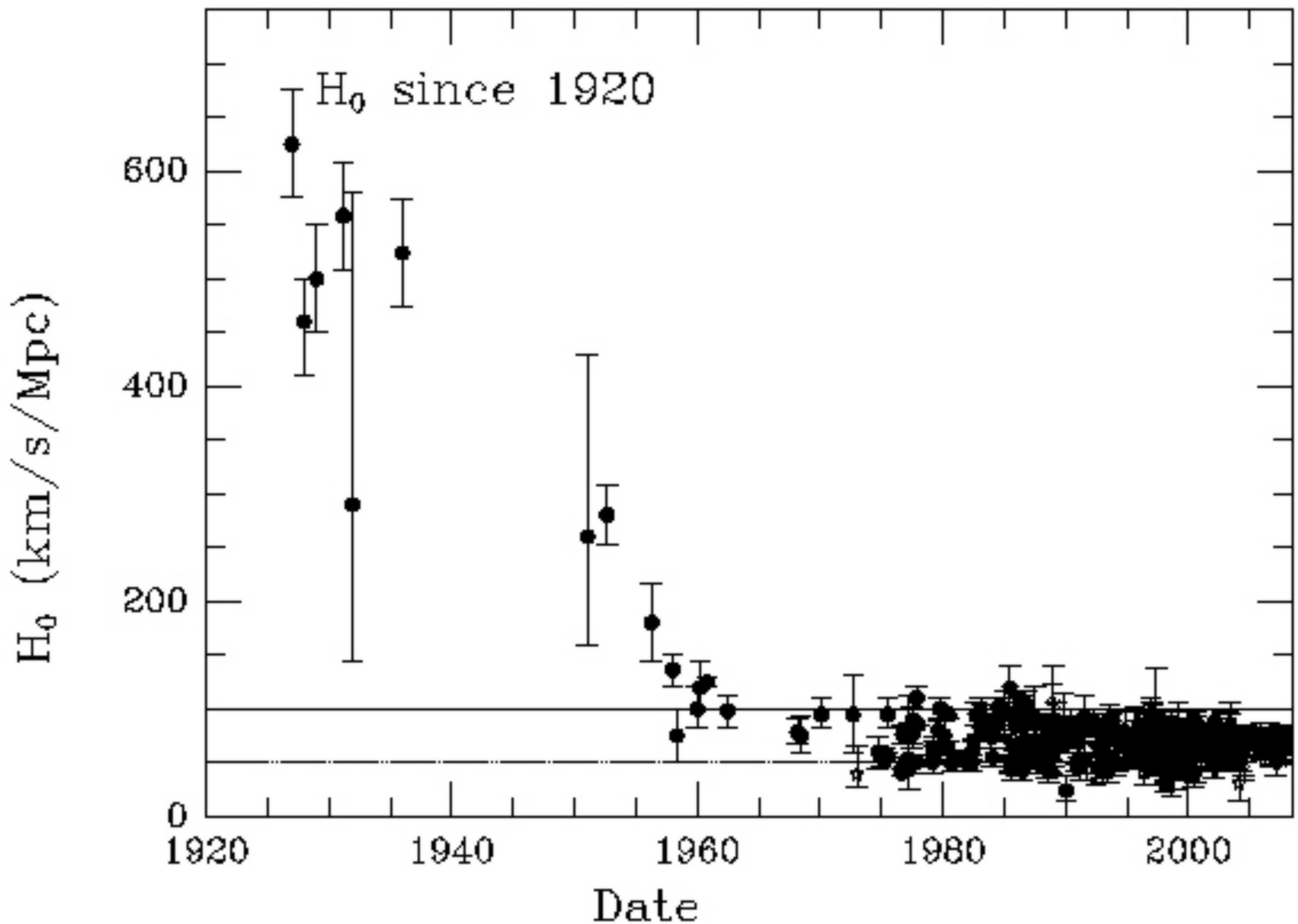
WHAT TO OBSERVE

You might think the easiest cosmological parameter to observe is the Hubble parameter (it was the first). And that we can determine the rest of the parameters from measuring H(z) over a good fraction of the Universe's history.

But it turns out the Hubble parameter is hard to measure, it is probably the most uncertain of the six parameters in the ΛCDM model.

HUBBLE HISTORY

If we look at measurements of H_0 over time, we see that they used to be almost an order of magnitude larger and the uncertainty has only fallen below 50% about 10 years ago.



Little h

In the 1980's there was an argument over the value of the Hubble parameter. One group found 100 Mpc/km/s while another found 50 Mpc/km/s. This factor of two was so persistent that a fudge factor, h, was introduced so that H_0 = 100h Mpc/km/s. Then you could just carry the h around in your calculations and not offend anyone. Many quantities are still given in terms of h today. If the h comes with a subscript, h_{50} or h_{70}, that means they used those numbers instead of 100 in the above relation.

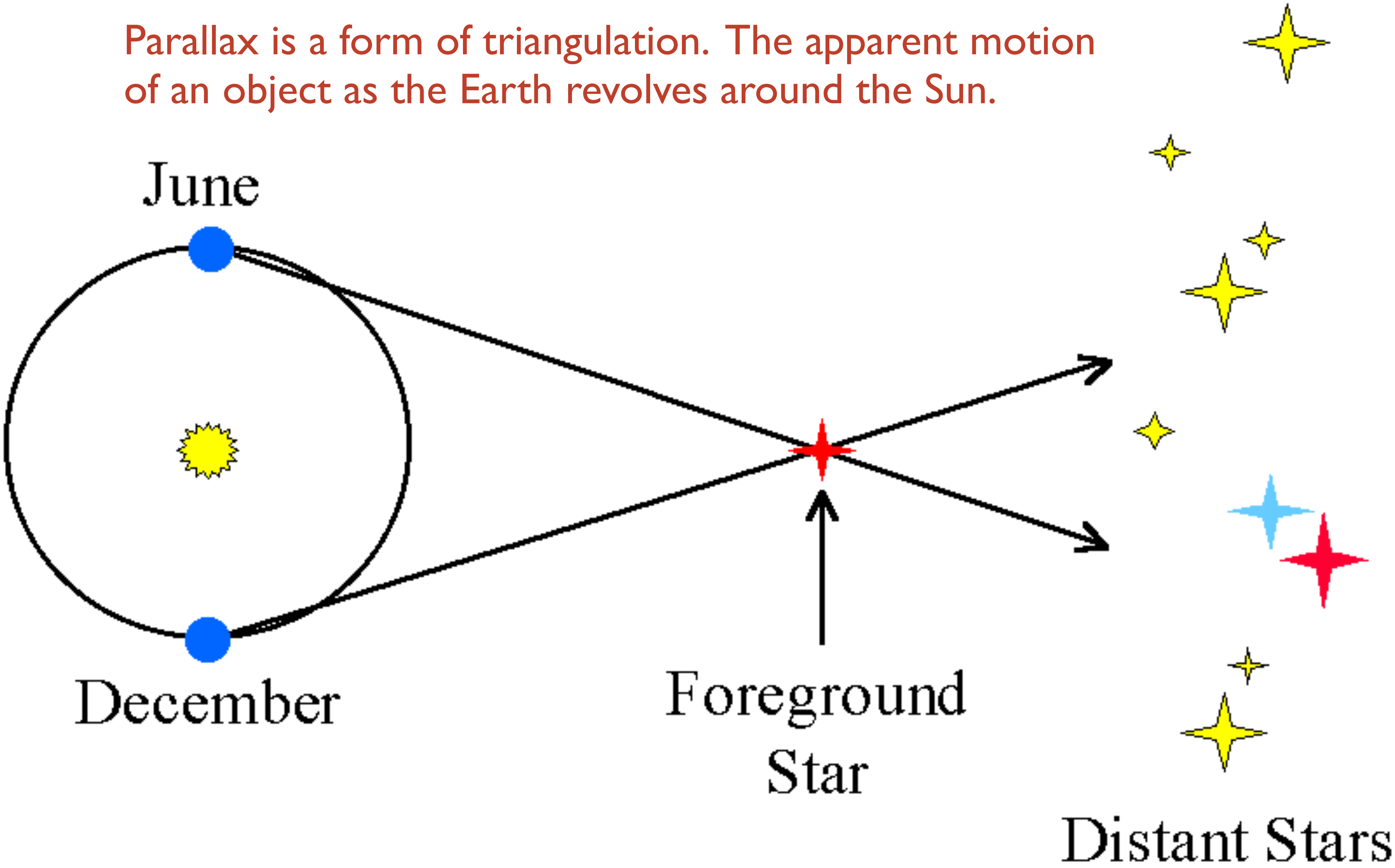
DISTANCE IN ASTRONOMY

That might seem rather strange, since to measure H_0 all one has to do is get a redshift, which can be done very precisely, and measure the distance.

Unfortunately, accurate distances in astronomy are rather problematic. The normal ways of calculating distance, rulers or light travel time, don't work outside our solar system. In fact distance is just about the hardest thing to measure in astronomy.

PARALLAX

Parallax is a form of triangulation. The apparent motion of an object as the Earth revolves around the Sun.



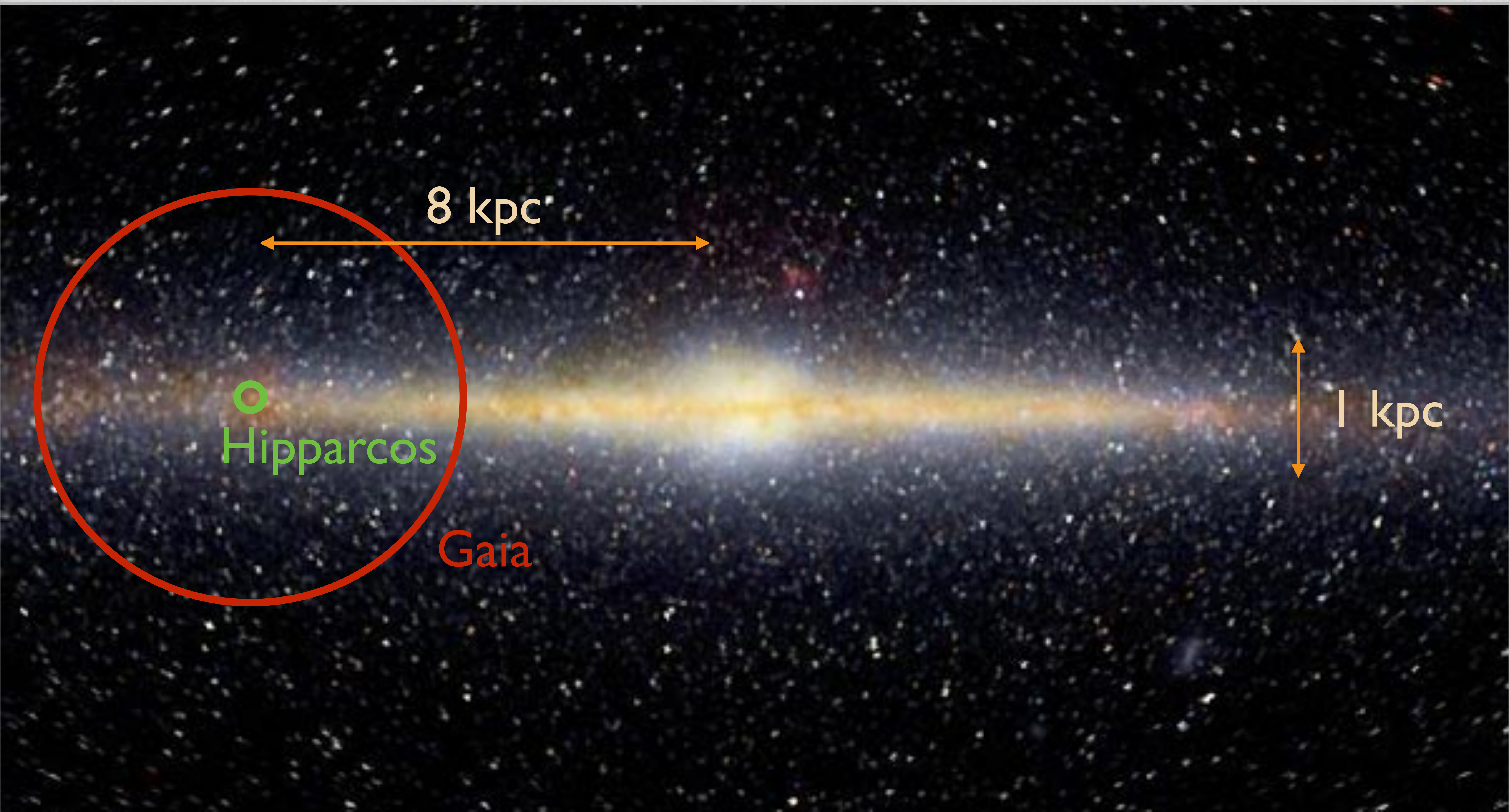
PARSEC

Parallax is where parsecs come from. 1pc is the distance an object is from Earth to have a parallax of 1 arcsecond. From this you can see that parallax can only be used when objects are pretty close to us, 1kpc would be a parallax of 1 milli-arcsecond and the nearest galaxies 1 Mpc would be 1 micro-arcsecond.

So no cosmological distances can be measured with parallax, that leaves us with standard candles.

Mission	Dates	angular error	max distance
Earth Telescopes		~0.1”	1pc
Hipparcos	1989-1993	~0.001”	100pc
Gaia	2013-2022	~2×10^{-5} ”	5kpc

Gaia measures parallaxes to about 1/4 of our Galaxy. This is a tremendous improvement.



STANDARD CANDELS

The main way distances are measured is astronomy is though standard candles, which means objects whose luminosity we believe we know. Then

$$F = \frac{L}{4\pi d_L^2}$$

where d_L is the luminosity distance to the object. The first standard candles discovered where variable stars. Today Type 1a supernova serve as the best standard candle at cosmological distances.

VARIABLE STARS

Two types of variable stars were discovered in the early part of the 20th century.

RR Lyrae stars and Cepheid variables.

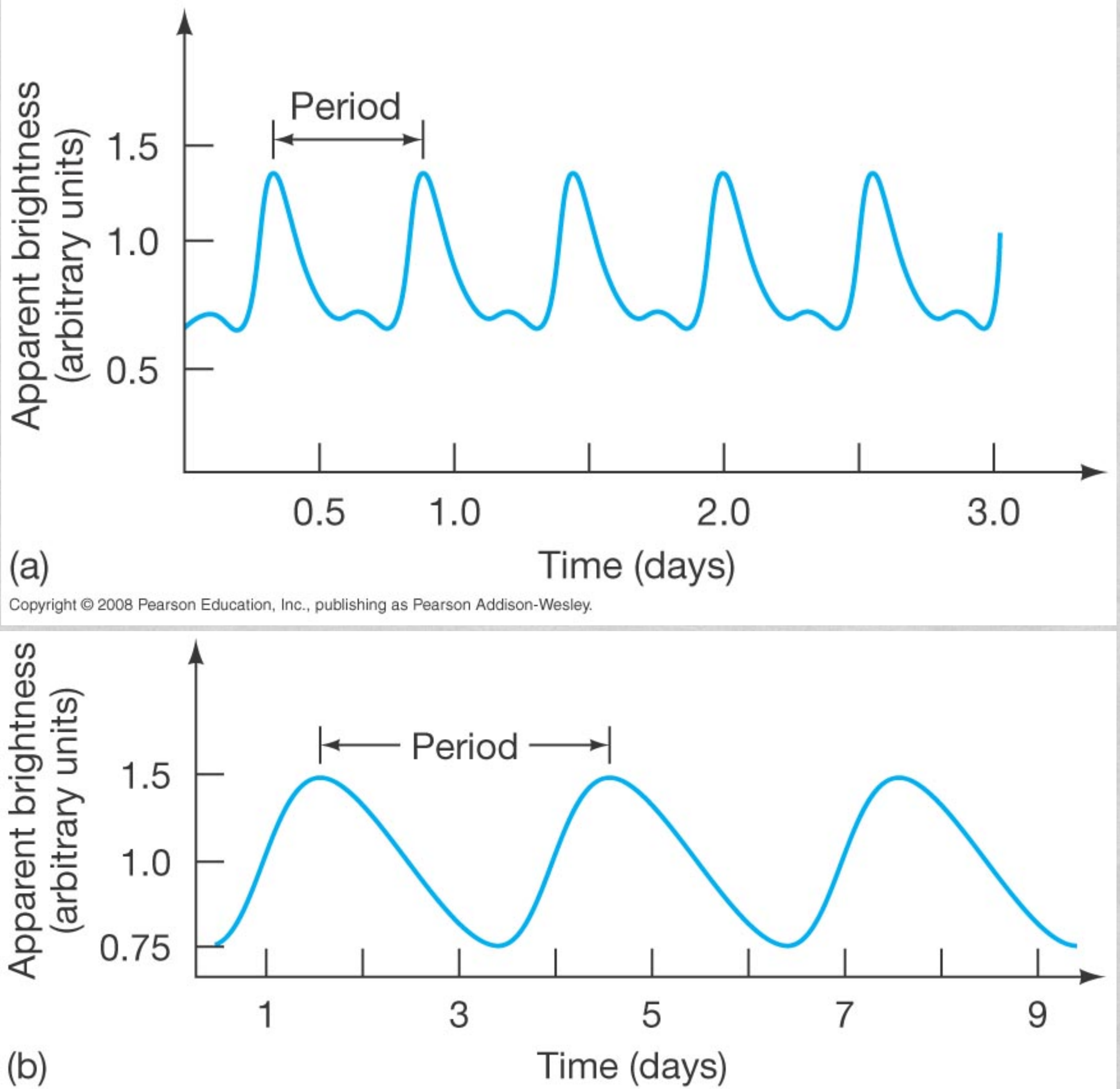
RR Lyrae have periods of that range form 0.5 to 1 day.

Cepheids have periods that range from 1 to 100 days.

Both types of stars change in brightness by about a factor of two, so they can easily be detected and identified.

Because only the apparent brightness is needed to identify them large regions can be searched for variable stars.

It was variable stars that first allowed Hubble to give distances to nearby nebula and show that they were actually distinct galaxies and then to derive the Hubble Law.



WHY TO THEY PULSATE?

The structure of a star is in large part determined by how easily radiation can pass through it.

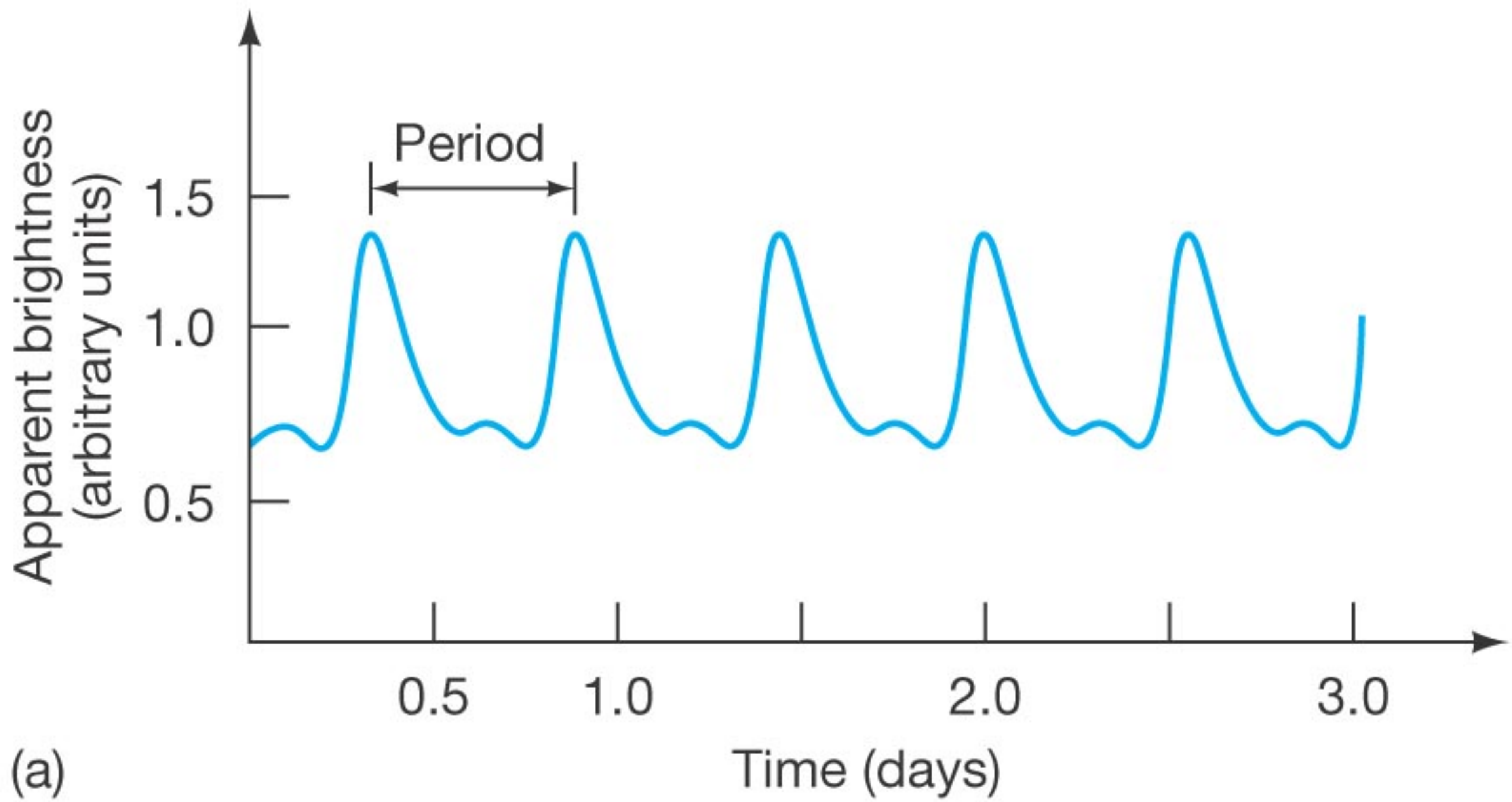
If the opacity of a star increases, the radiation will become trapped and puff out the star. If the opacity falls the radiation escapes more easily and the star shrinks.

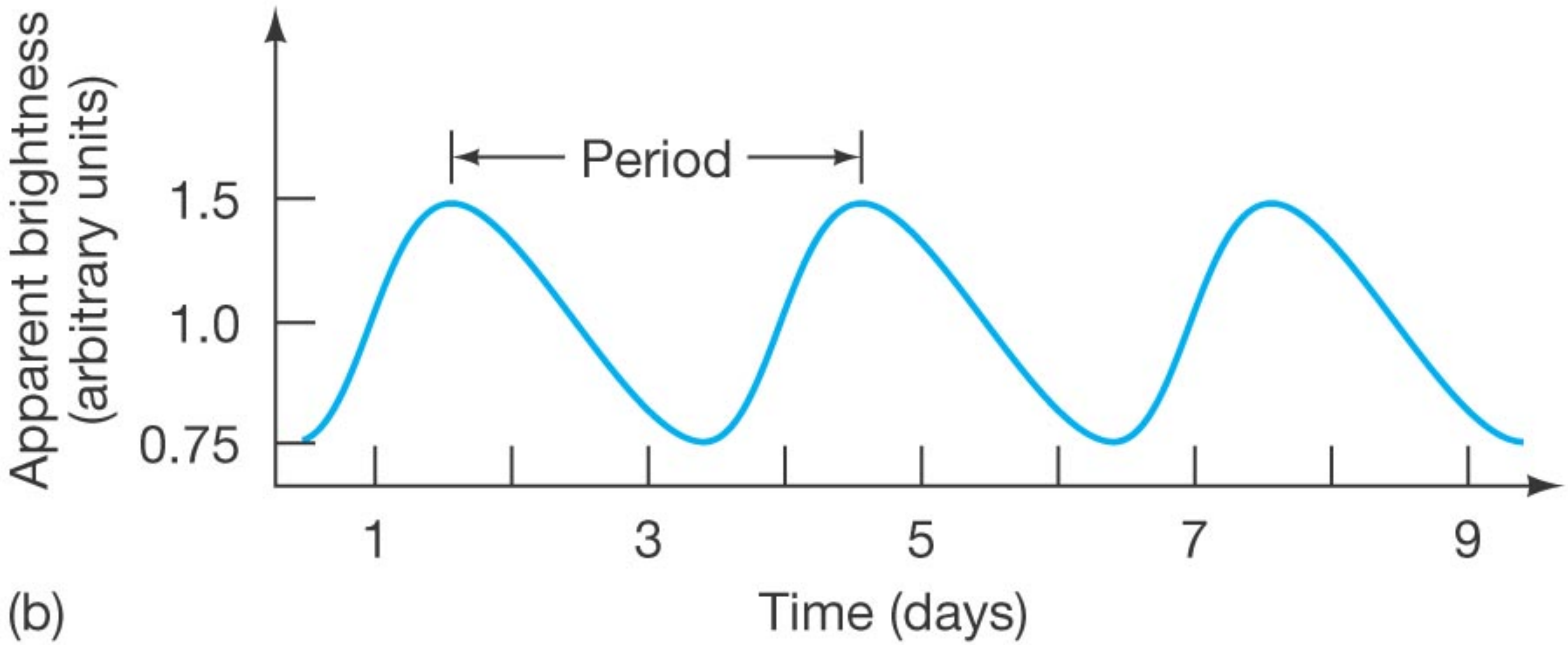
Variable stars have atmospheres where a little cooling decreases the opacity by a lot causing them to shrink and heat up. Heating up increases the opacity causing them to expand and cool.

Pulsation due to feedback loop:

An increase in T

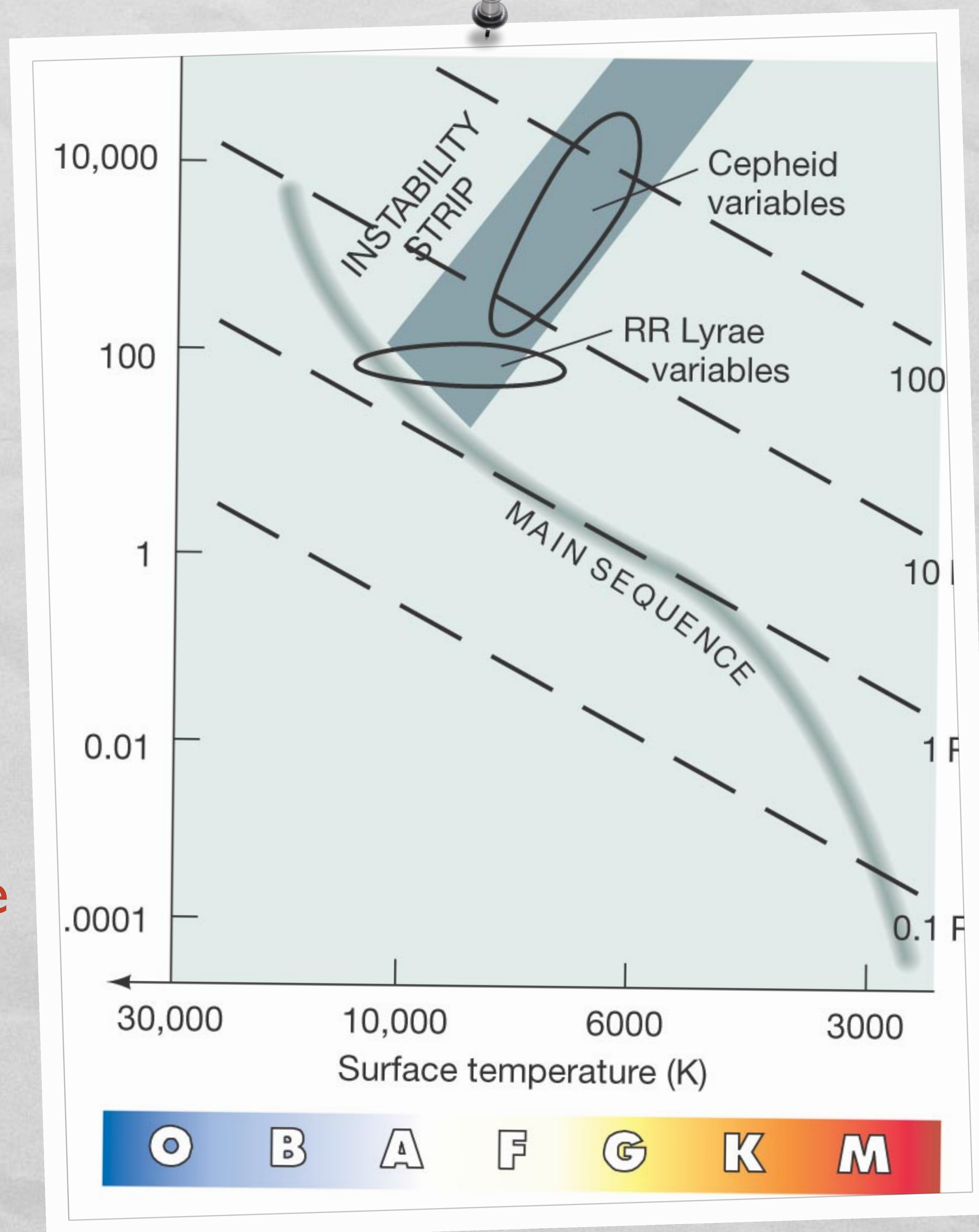
- HeIII (doubly ionized, He^{2+})
- high opacity
- radiation can't escape
- even higher T and P
- atmosphere expands
- lower T
- HeII (singly ionized, He^{+})
- low opacity
- atmosphere contracts
- repeat…

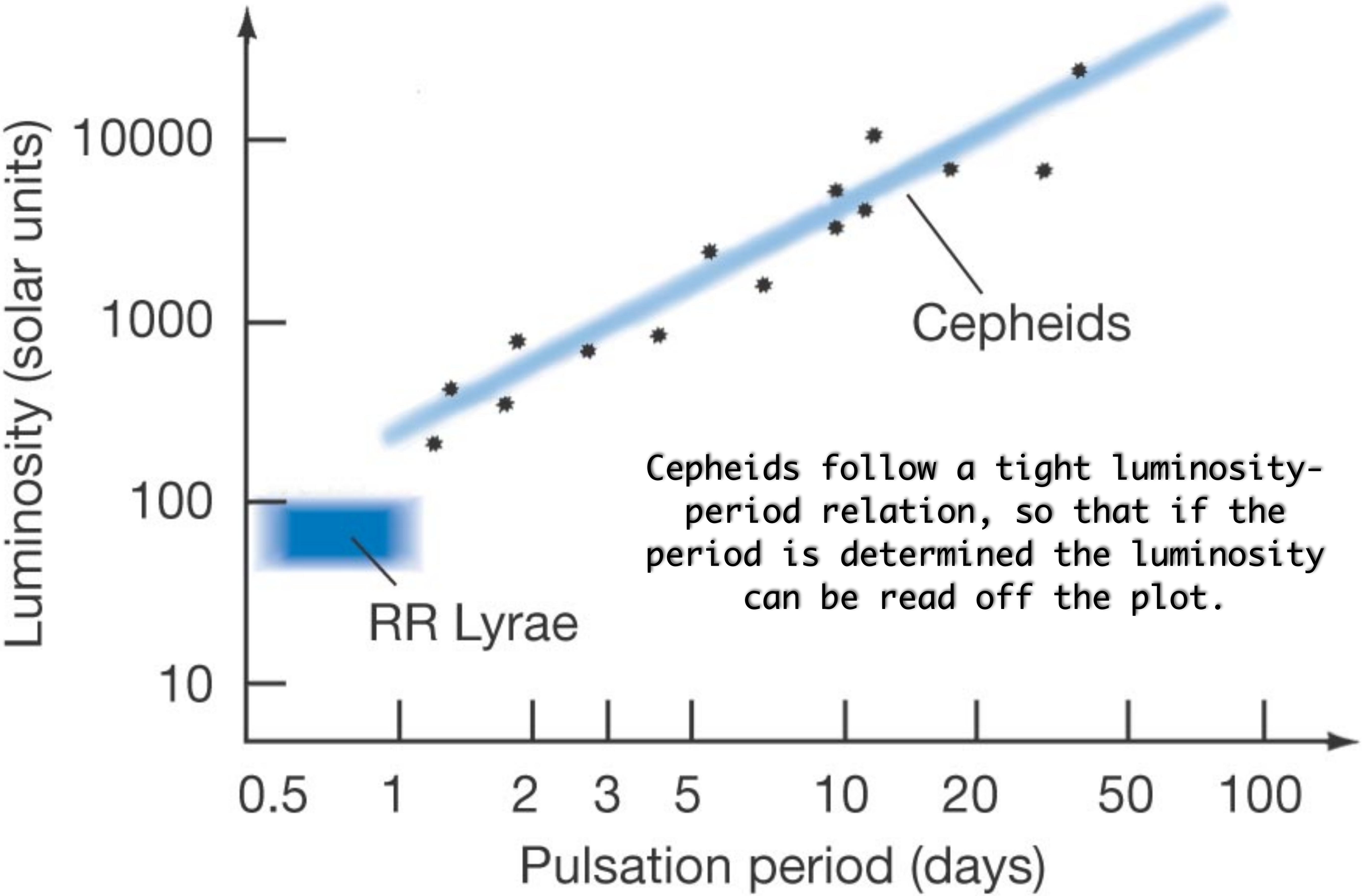




These instabilities do not happen to normal stars, but to evolved stars in the red giant phase.

Stars that pass into this instability strip pulsate for about a million years till they move to a different region of the H-R diagram.





Cepheids follow a tight luminosity-period relation, so that if the period is determined the luminosity can be read off the plot.

Aaronson et al

Mould et al

HST Key Project

One of the key projects of HST was an attempt to improve all these measurements using HST to get H_0 to 10%.

This flowchart shows the number of steps in getting the distance to relatively nearby galaxies.

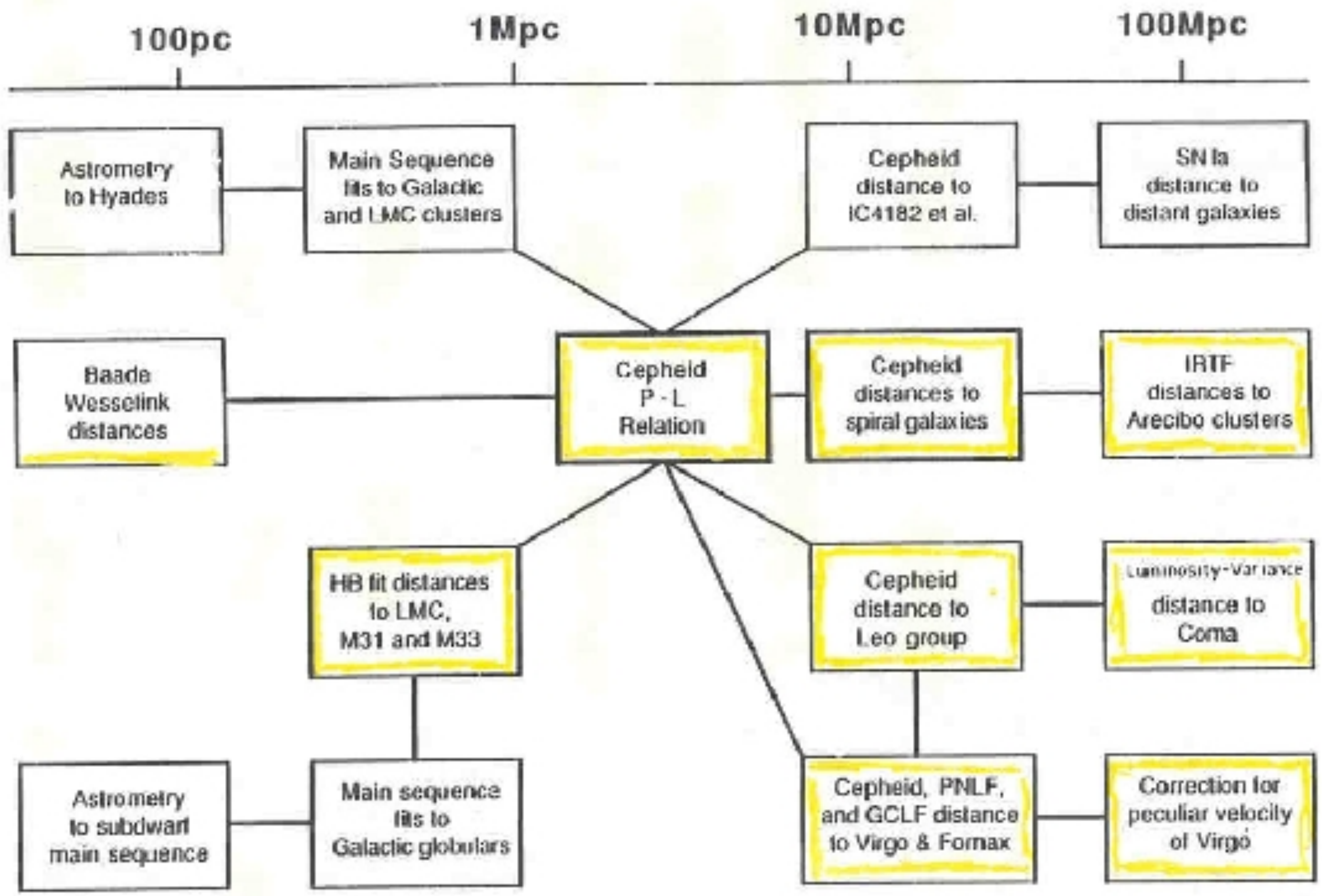


Figure 2

In this program the Cepheid P-L relation is calibrated within 1 Mpc and applied to spiral galaxies at typically 10 Mpc. These measurements in turn calibrate secondary distance indicators which reach out to 100 Mpc.

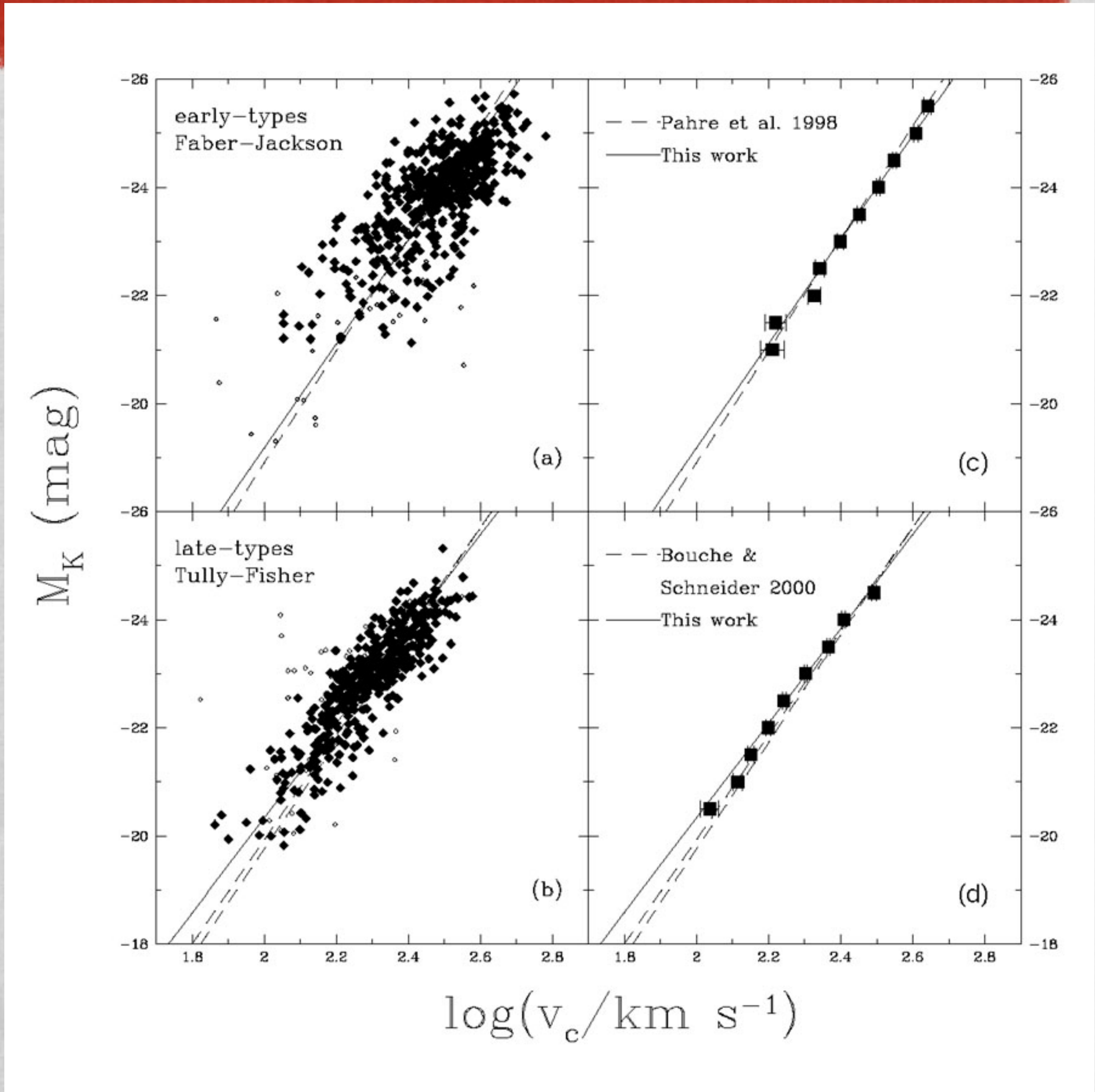
OTHER CANDELS

Many other standard candles have been used over the years. The most important of these are the Tully-Fisher and Faber-Jackson relationships. These are observations that a galaxies rotational velocity (for disk galaxies) or velocity dispersion (for ellipticals) are related to their luminosity, thus enabling them to be used as standard candles.

TULLY-FISHER

The Faber-Jackson relation is not that precise, a more accurate version adding a galaxy's size is called the fundamental plane.

Tully-Fisher measurements are more accurate but still of order 50% error.



TULLY-FISHER

Why does such a relation exist? Well kind of obviously the more massive a system is the brighter it should be and the more gravity it will have.

A dark matter halo is usually defined by having an average density of some value Δ_c greater than the universe's mean density. So then the mass is given by

$$M = \frac{4}{3}\pi R^3 \Delta_c \bar{\rho}_c$$

A reasonable density profile is the singular isothermal sphere (SIS) which has a flat rotation curve.

$$M(R) \propto R \rightarrow v = \sqrt{\frac{GM}{R}}$$

The first equation tells us the total mass goes like the radius cubed so

$$v^2 = GM\left(\frac{3M}{4\pi\Delta_c\bar{\rho}_c}\right)^{\frac{1}{3}} = \left(\frac{3G^3}{4\pi\Delta_c\bar{\rho}_c}\right)^{\frac{1}{3}} M^{\frac{2}{3}}$$

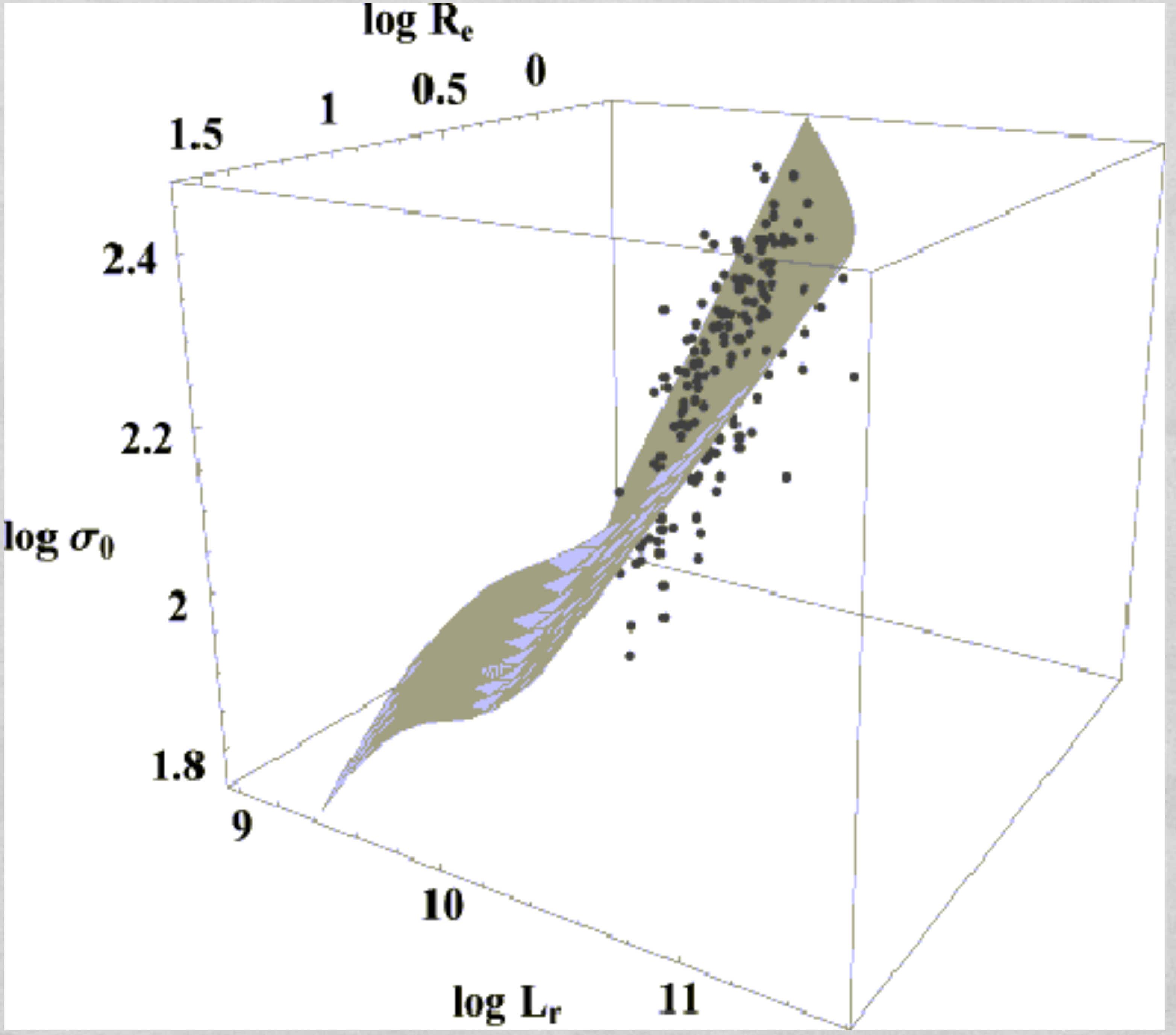
$$M = \left(\frac{4\pi\Delta_c\bar{\rho}_c}{3G^3}\right)^{\frac{1}{2}} v^3$$

TULLY-FISHER

If luminosity is proportional to mass we would thus expect a Tully-Fisher relation like $L \propto v^3$. The observed relation depends on the waveband used to measure the luminosity and the exponent is around 4 with lower values for IR bands and higher for UV. So we see IR light is more proportional to mass while shorter wavelength light which traces star formation is less directly connected to total mass.

FUNDAMENTAL PLANE

For elliptical galaxies are much better fit is found if one considers the size of the galaxy along with its velocity dispersion and luminosity.

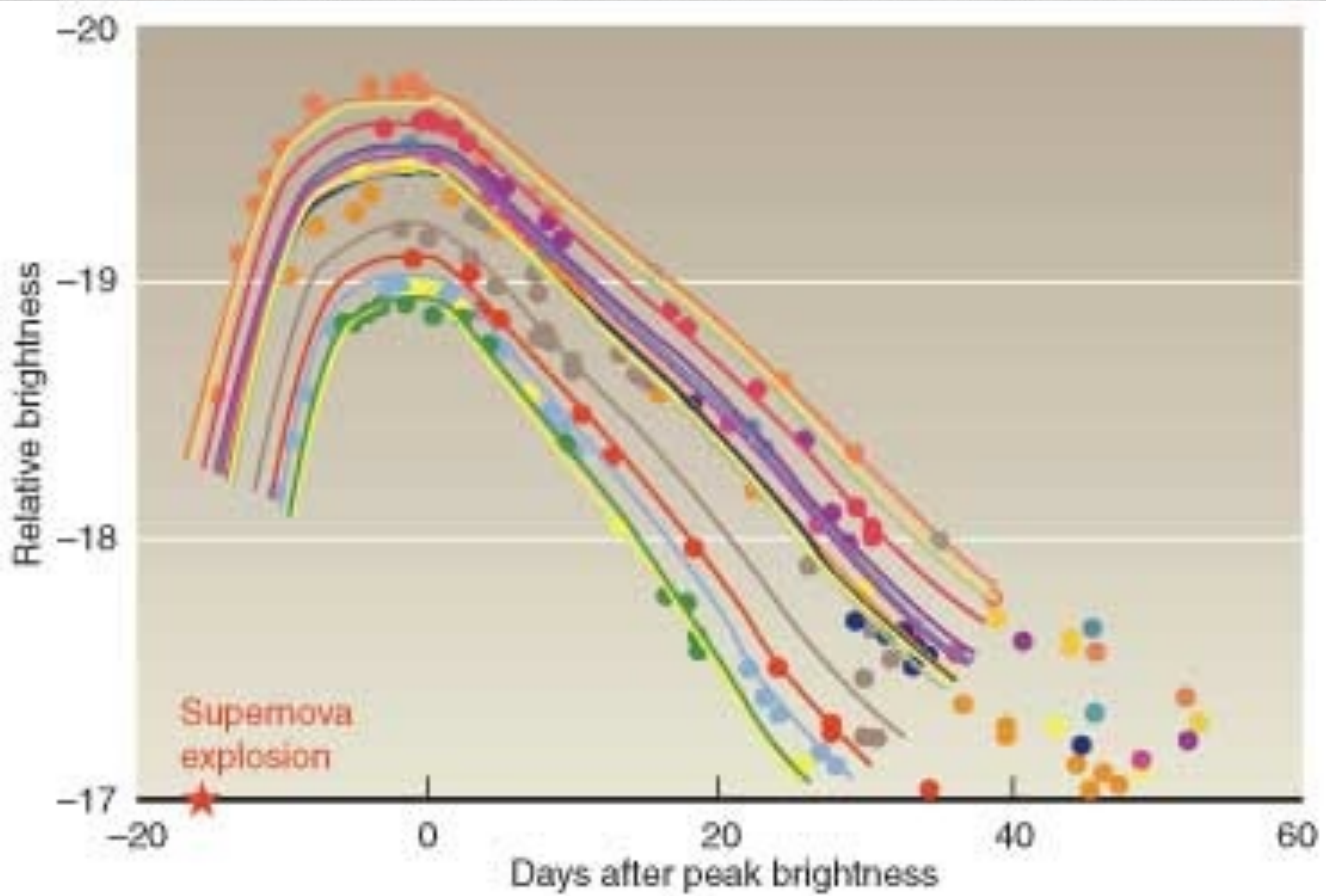


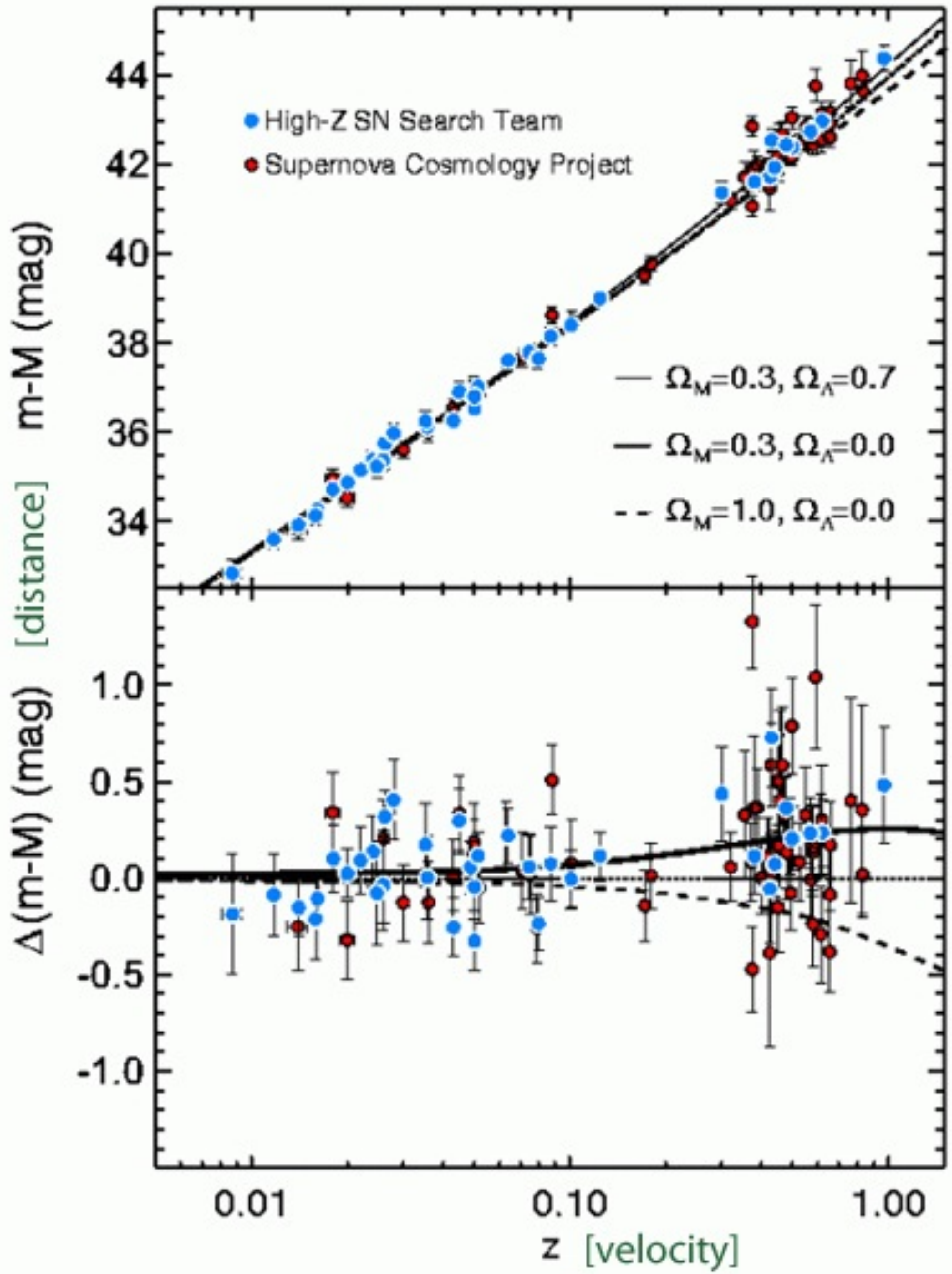
TYPE 1A SUPERNOVA

By far the best standard candle today are Type 1a supernova, the explosions of white dwarfs. Since all white dwarfs have the same maximum mass ($1.44M_\odot$) it makes some sense that the explosion should have similar luminosities.

However, theoretically we don't understand why these should be such good standard candles. In general explosions are messy and not good standards.

However, they seem to show a remarkable uniformity that can be improved by studying their light curves. Like the variable stars, the time it takes a Type 1a supernova to decay tells you its peak luminosity.





While Type 1a supernova make excellent standard candles, they actually aren't that good for measuring H0.

This is because supernova are very bright, but also rare. It is hard to find many nearby.

However, supernova at moderate and high redshift gave us our first evidence that H(z) increasing, not slowing down.

THE DISTANCE LADDER

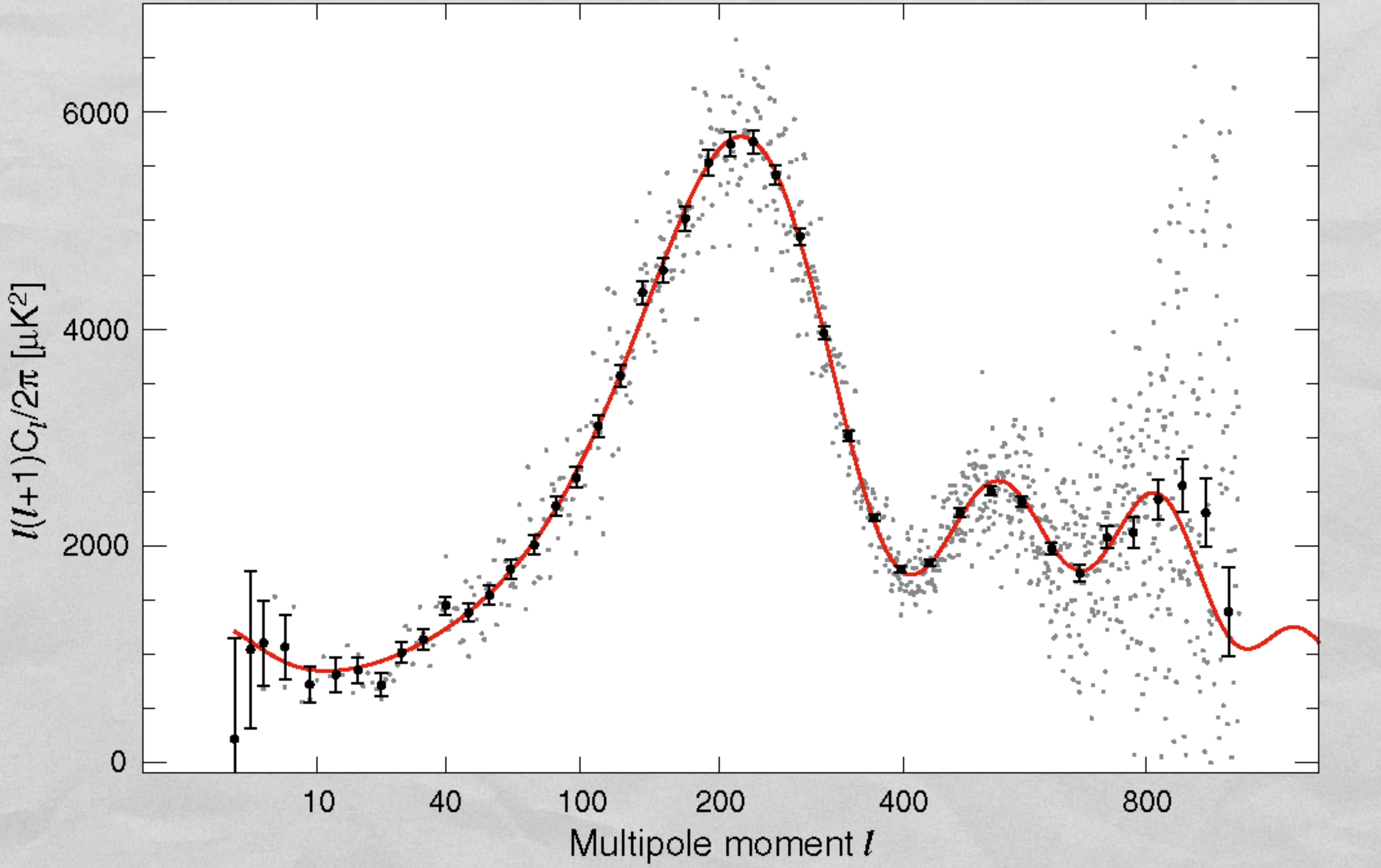
Method	Scatter	Reach	Systematics
Parallax	~d	<1 kpc	
Cepheids	5-10%	30 Mpc	Metallicity
SBF	5-10%	50 Mpc	Stellar LF
Tully-Fisher	10-20%	>100 Mpc	Mass-to-light
FP/D_n-sigma	10-20%	>100 Mpc	Kinematics
SN Ia	5-10%	>1000 Mpc	Dust

STANDARD RULERS

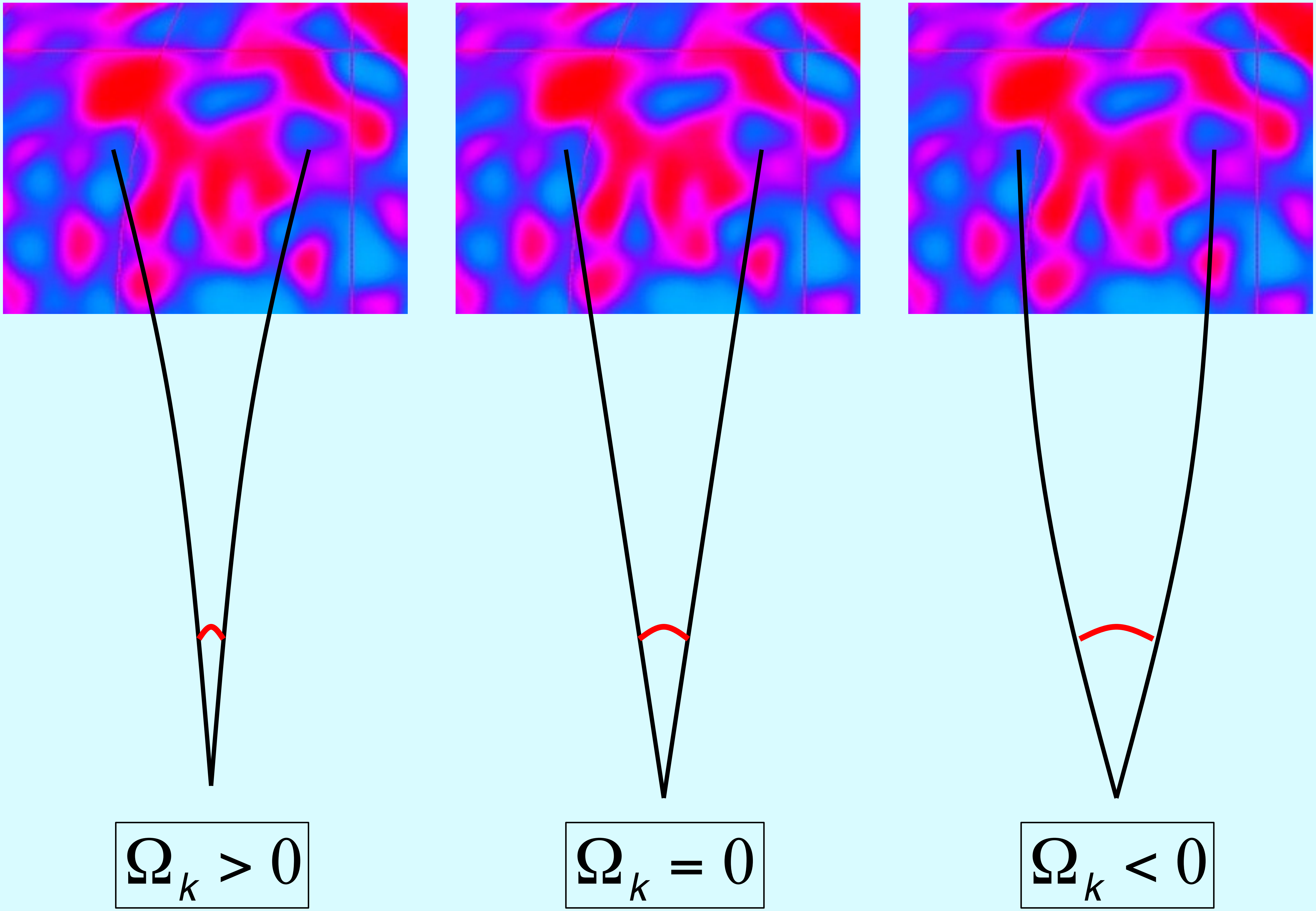
An alternative to standard candles are standard rulers. If we know how big something actually is, then measuring its angular size tells us its angular diameter distance.

The problem is finding a standard ruler. The size of radio jets from radio galaxies is one that has been tried and works pretty well. But the most important one is from the CMB.

Fluctuations as we've seen occur at all scales. There is a special scale in the CMB. Sound waves in the baryon-photon fluid travel at the sound speed. If a perturbation is exactly the speed of sound times the age of the Universe (the sound horizon) then the baryons can maximally react to that perturbation. That value just depends on known physics so we can calculate where the physical scale of the first peak.



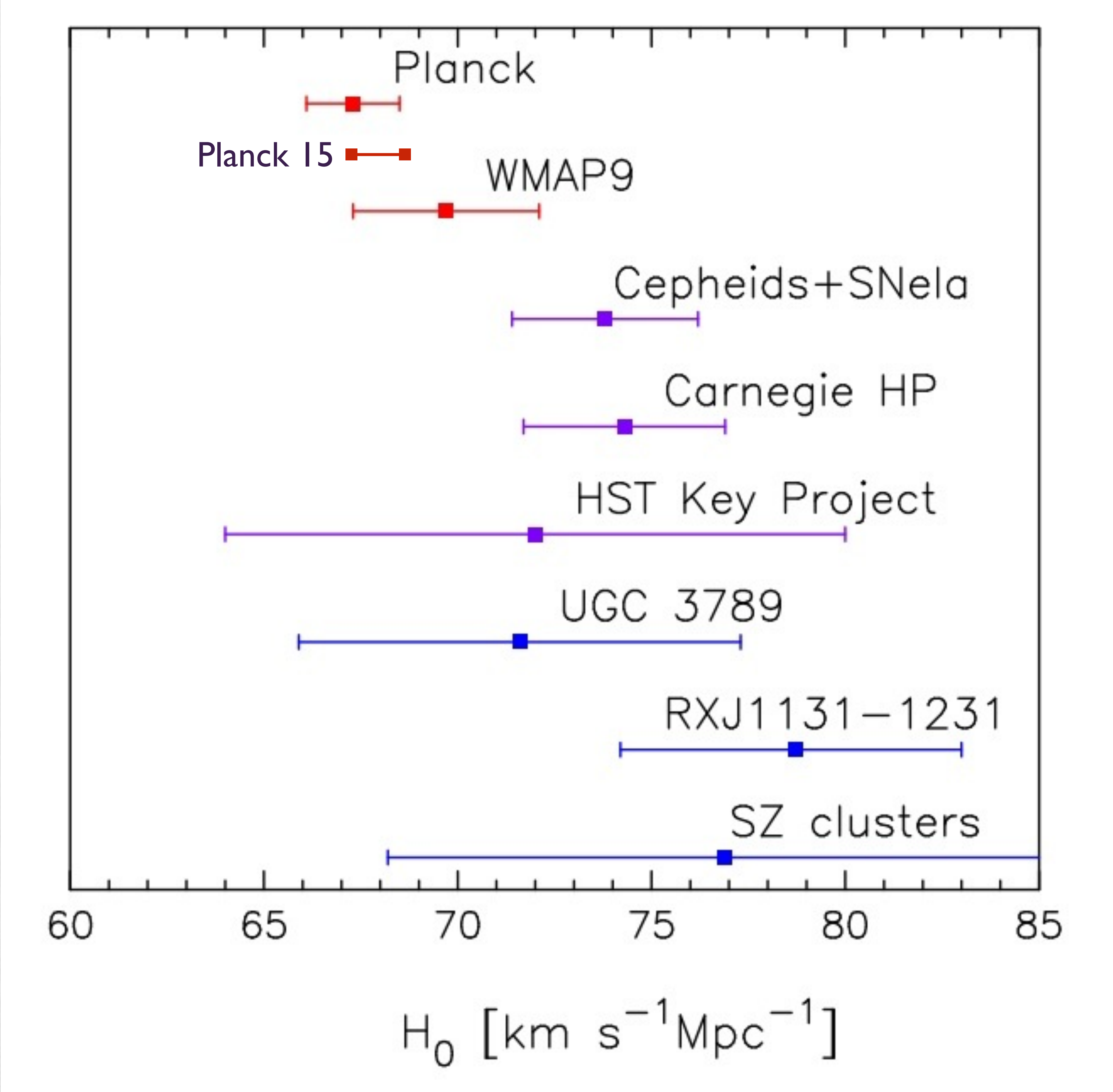
Constraining Cosmological Parameters



The current status is that the standard ruler and standard candle methods are giving slightly discrepant results.

This is maybe a little bit more than 1 sigma but already there are tons of papers trying to explain it.

The Planck 2015 value is slightly higher, but not much, so this tension remains.



REDSHIFT-SPACE DISTORTIONS

Besides the Hubble flow, galaxies will also move because of the gravitational pull of other objects. This gives each galaxy a peculiar velocity. When galaxy velocities are measured one gets both the Hubble flow velocity and the radial component of the peculiar velocity.

Hubble flow - H_0 d

radial component of peculiar velocity - v_r

Total radial velocity - H_0 d + v_r

REDSHIFT-SPACE DISTORTIONS

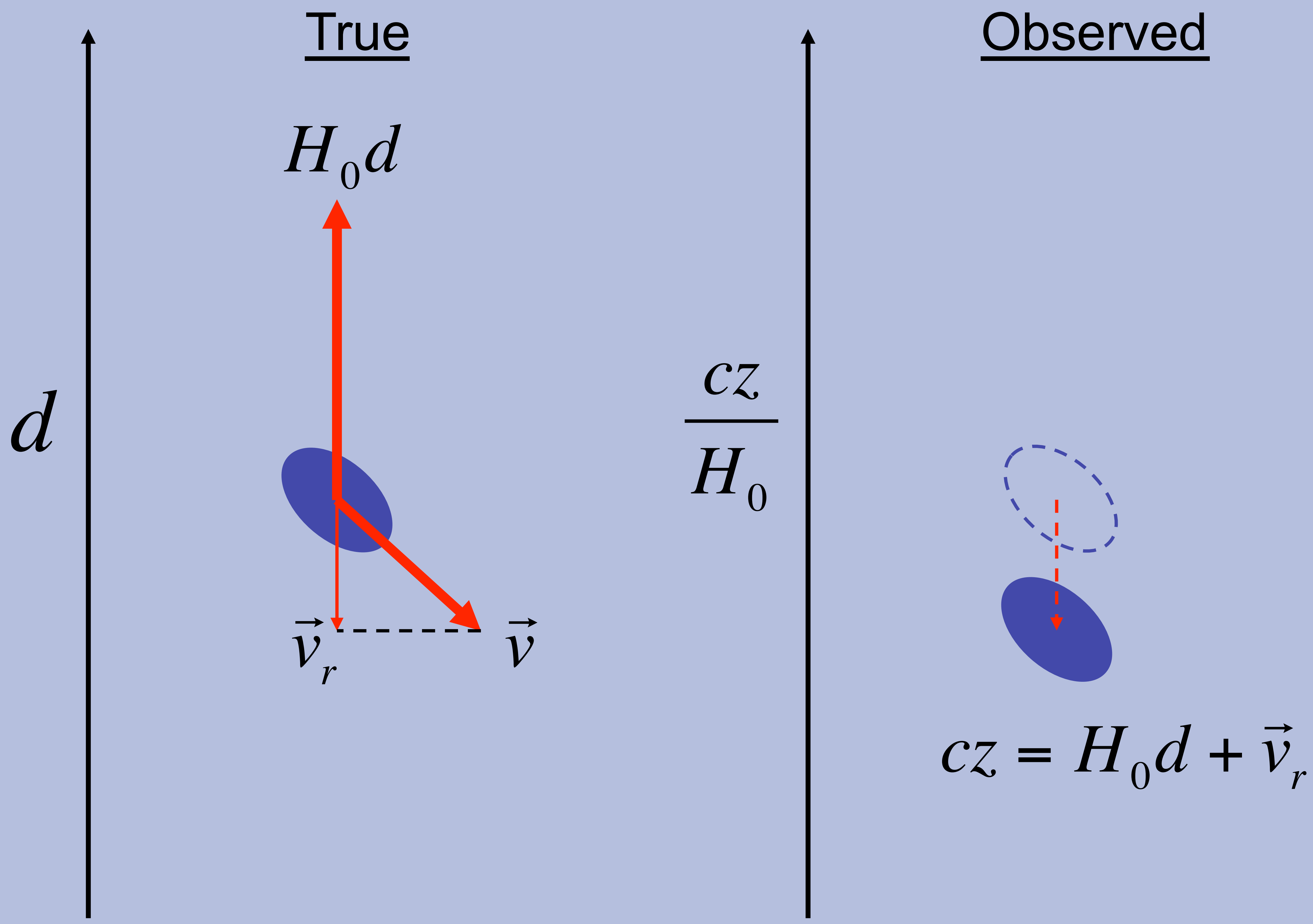
When we measure a redshift we can't tell what part of the velocity is cosmological and what part is peculiar. What we measure is the redshift z and infer a velocity v=cz. We infer the distance

$$d = \frac{cz}{H_0}$$

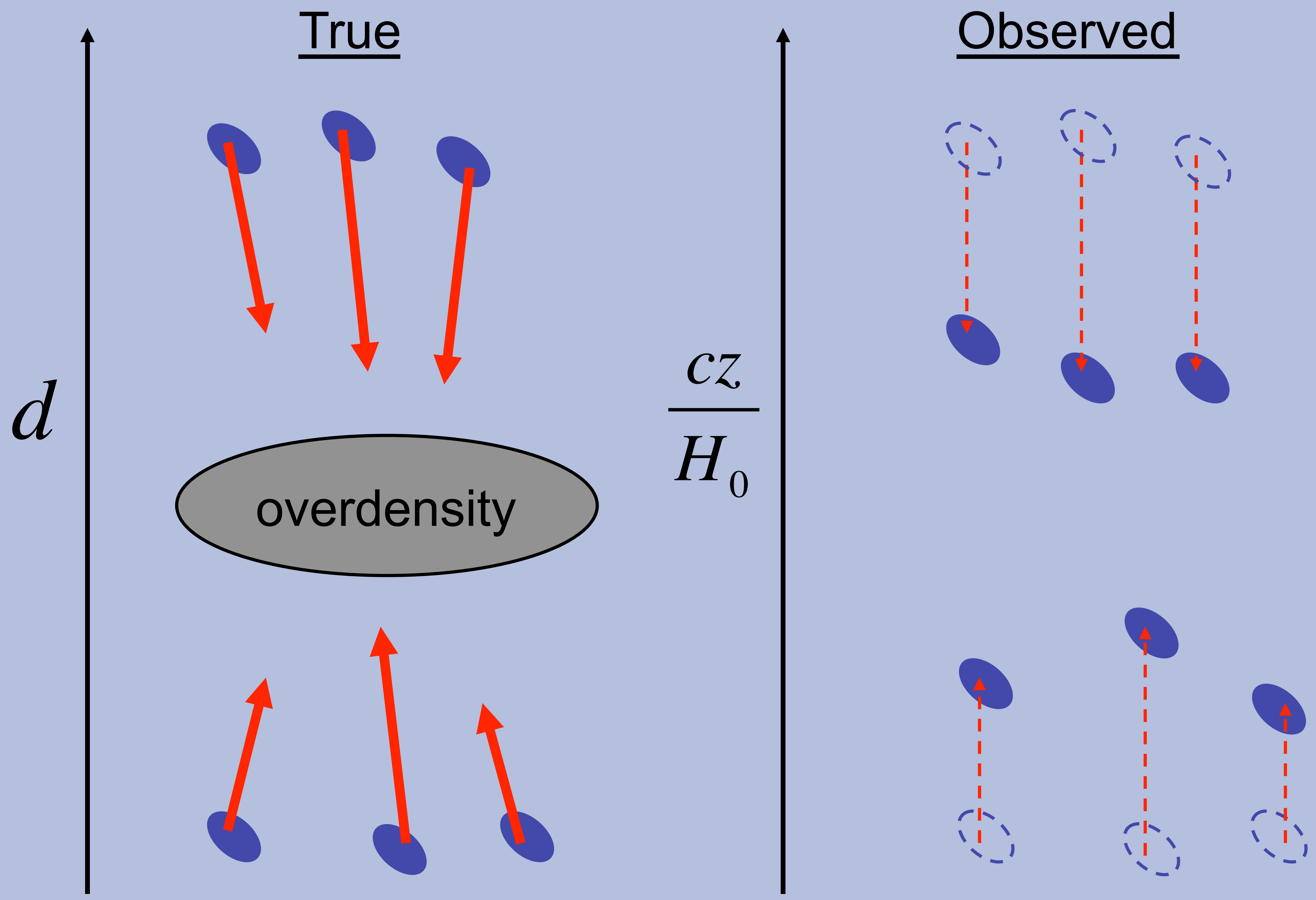
but really

$$\frac{cz}{H_0} = d + \frac{v_r}{H_0}$$

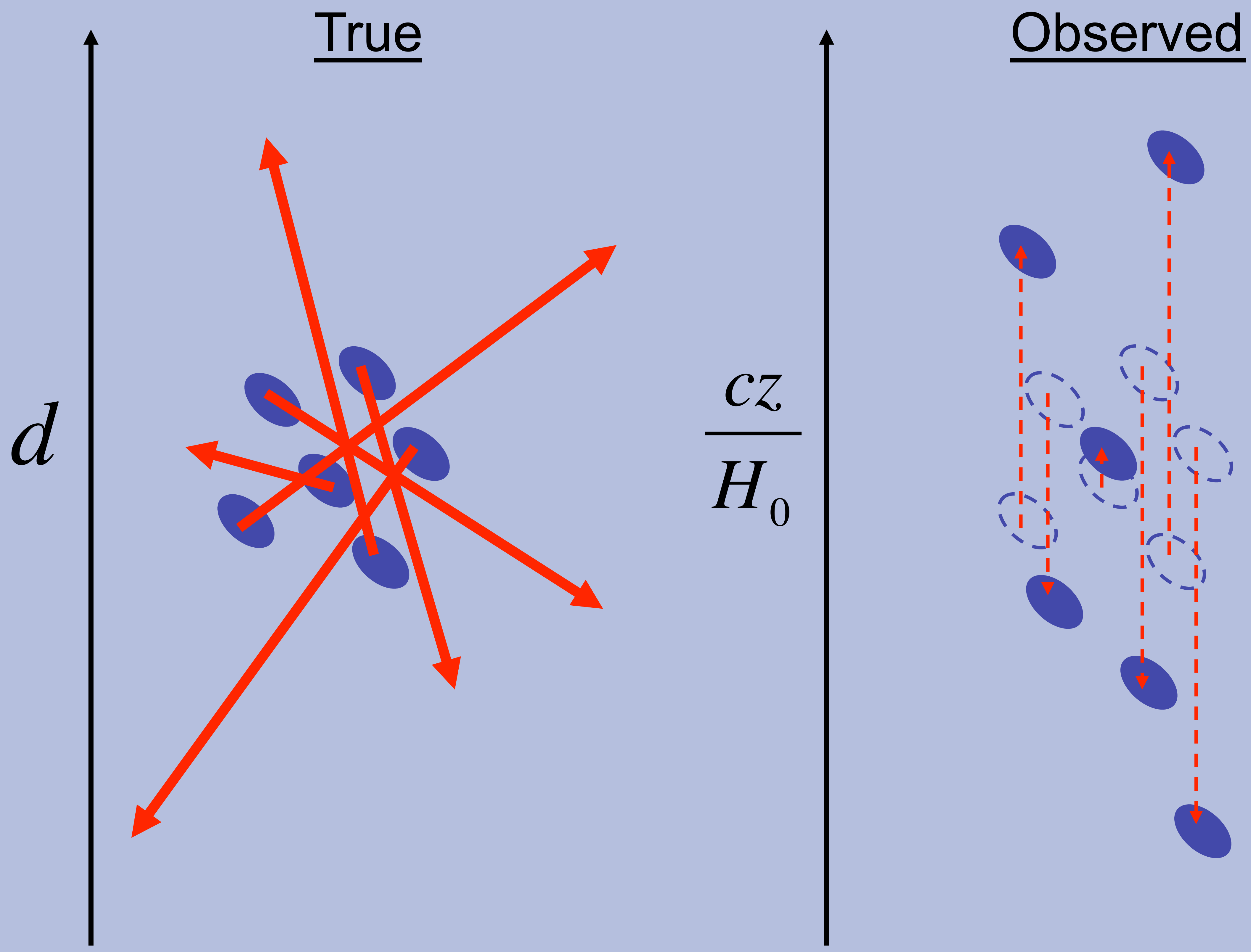
redshift-space distortions



redshift-space distortions



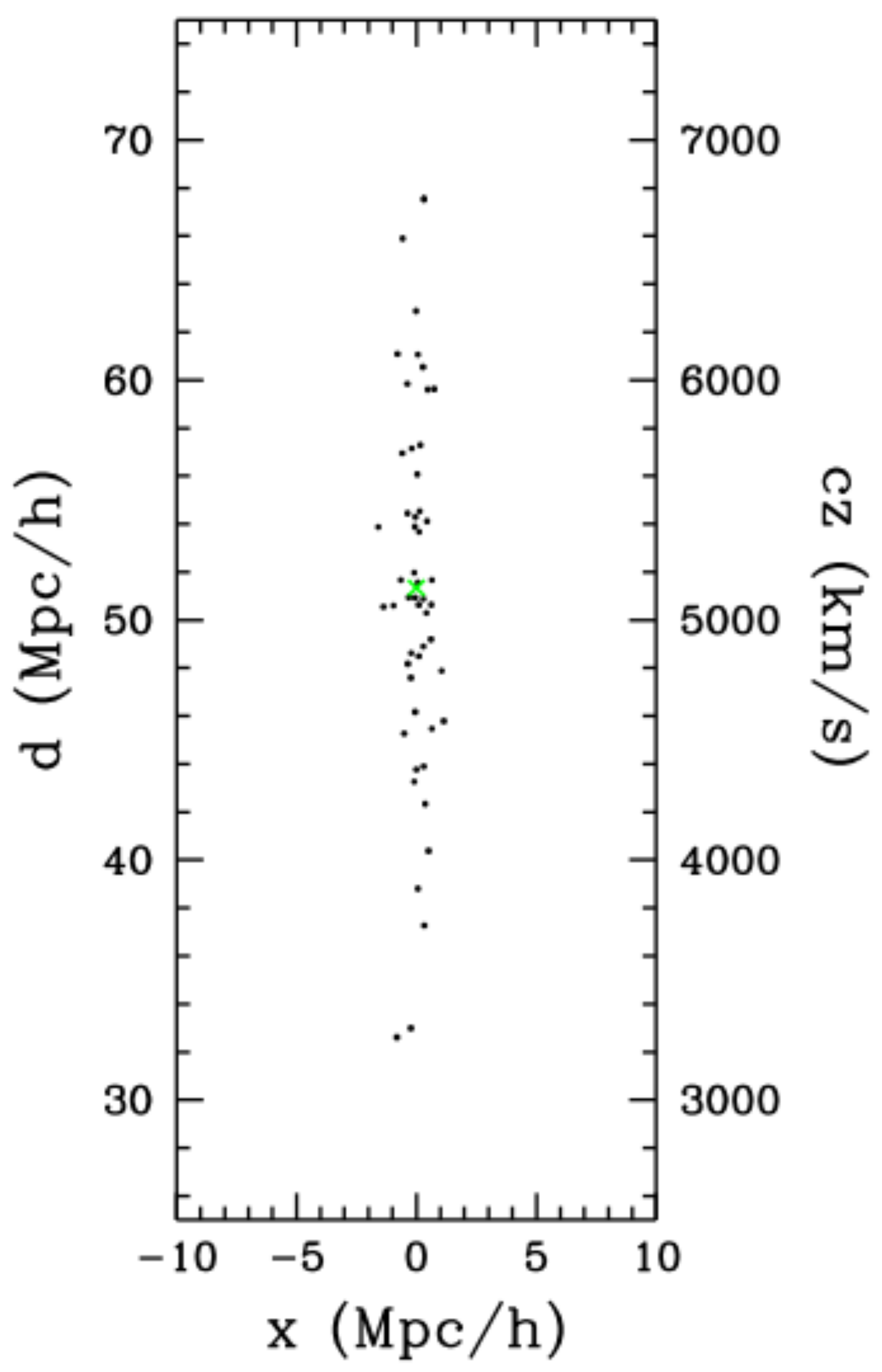
redshift-space distortions



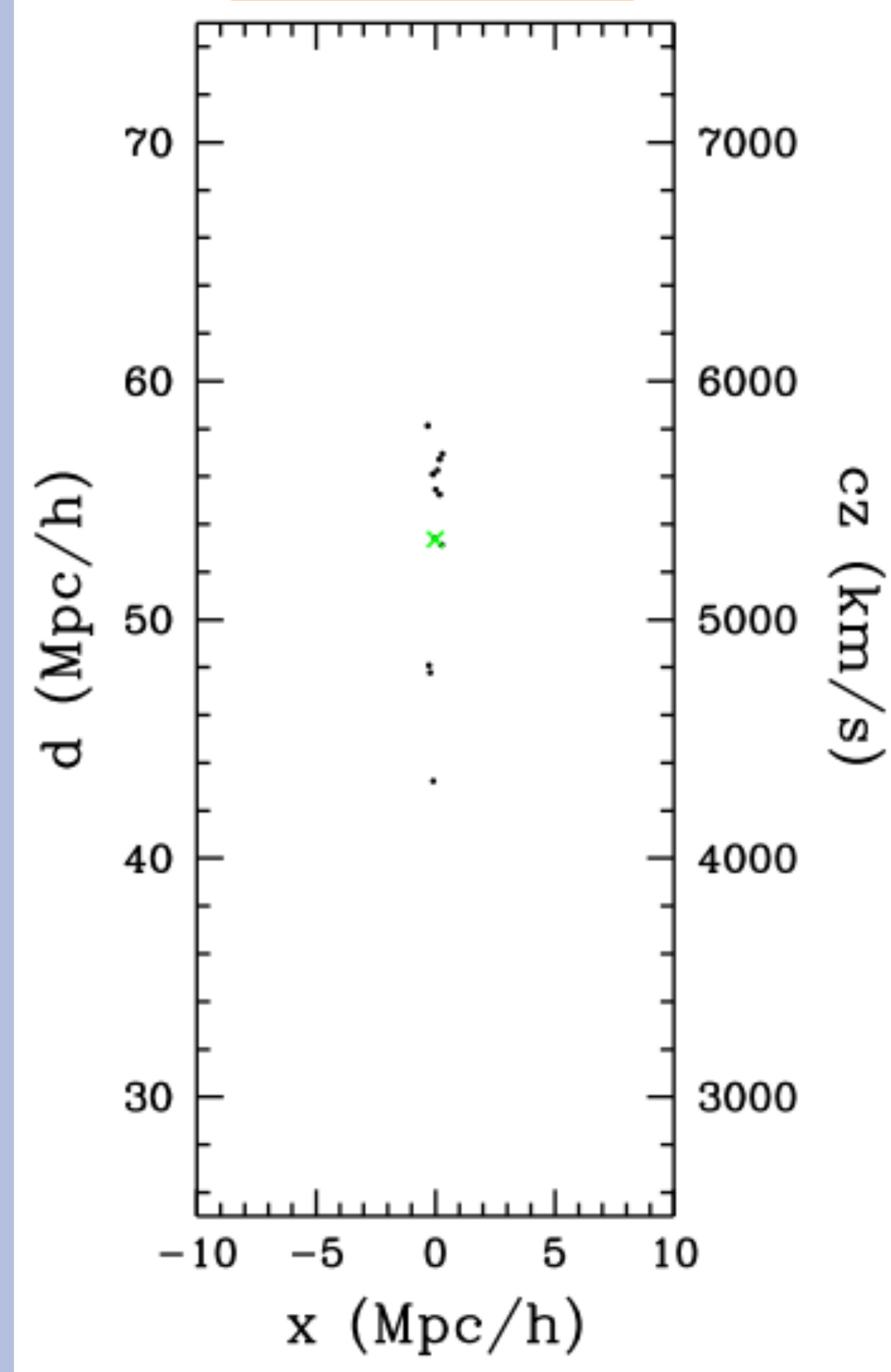
redshift-space distortions

real-space

$9.9 \times 10^{14} M_{\odot}$



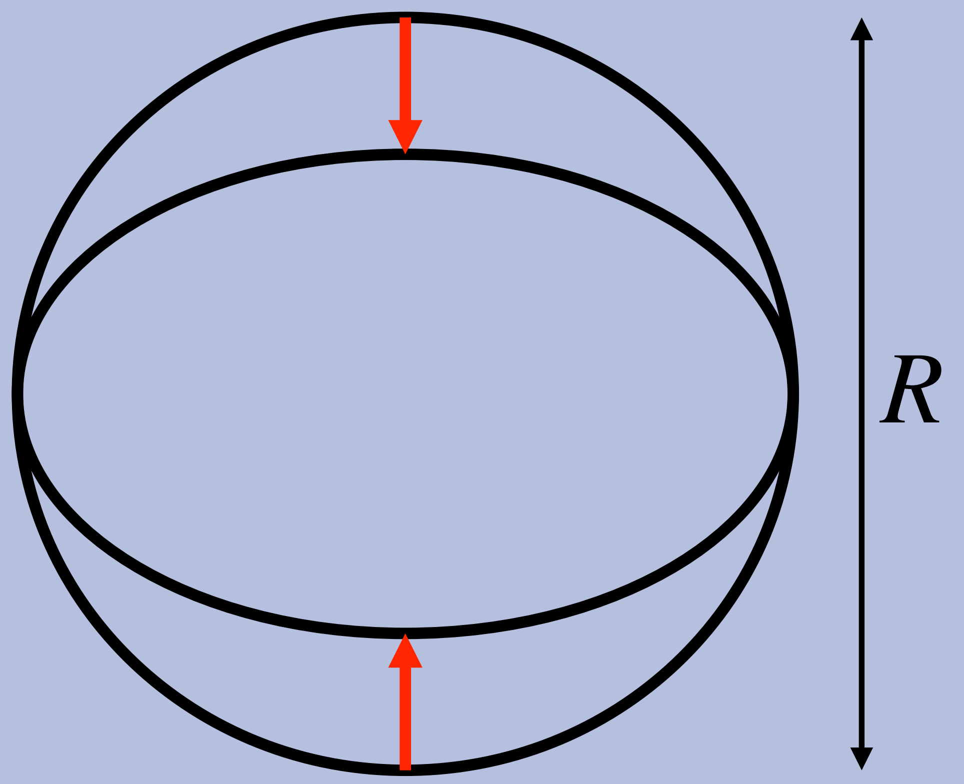
$9.5 \times 10^{13} M_{\odot}$



redshift-space distortions

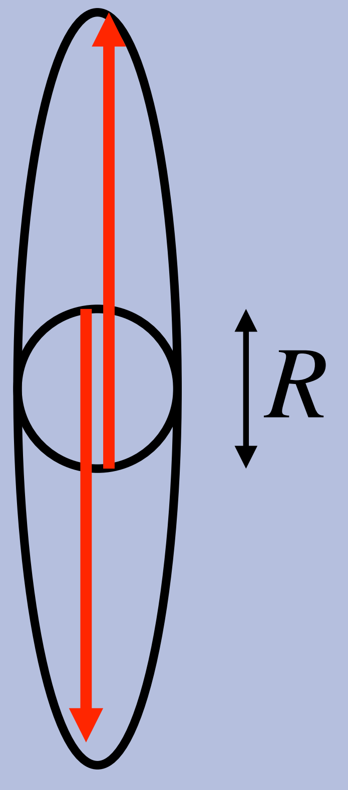
• Large scales: compression

$$H_0 R > \langle v_{\rm pec} \rangle$$



• Small scales: smearing (fingers of God)

$$H_0 R < \langle v_{\rm pec} \rangle$$

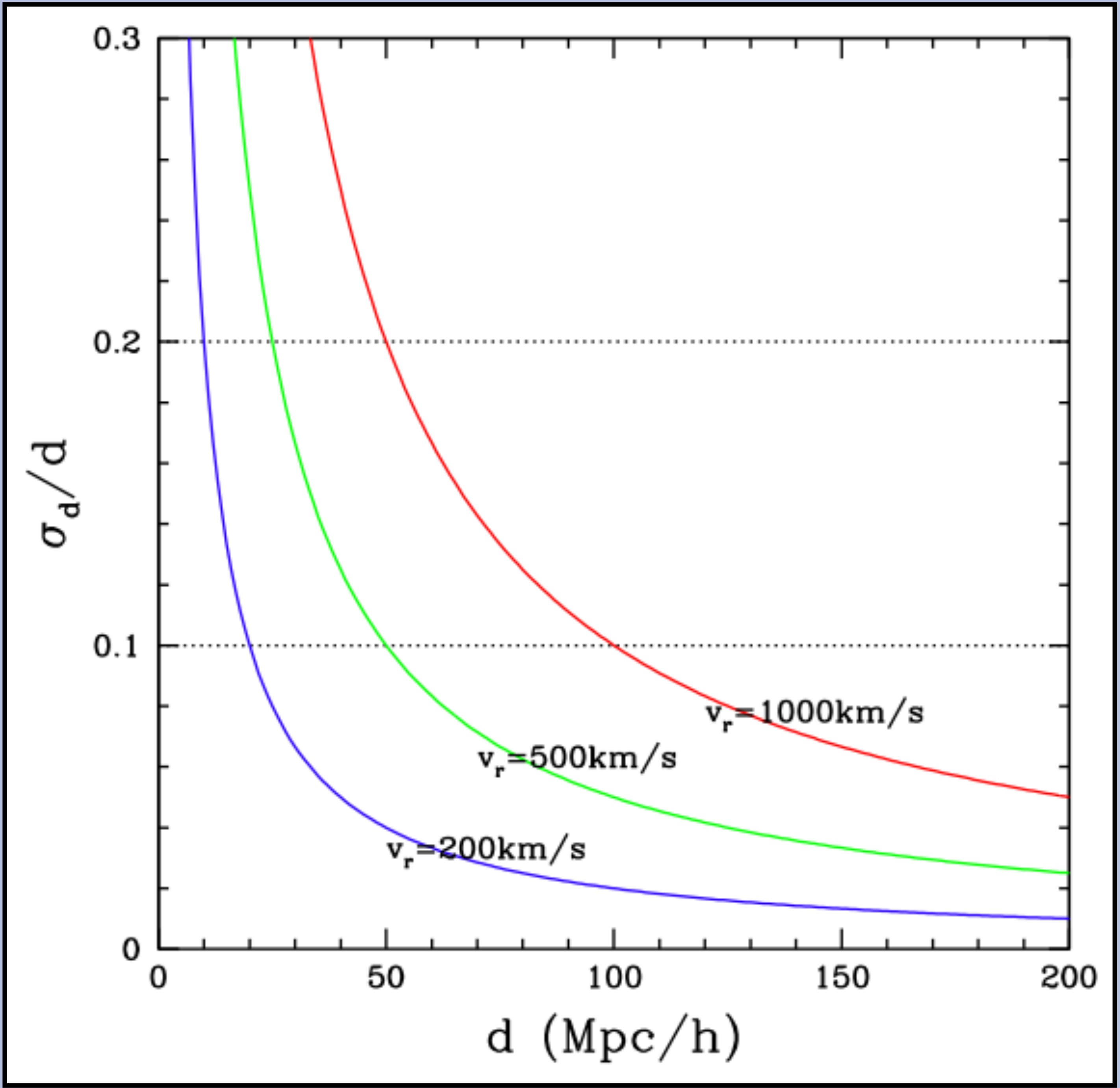


Redshift as distance estimator

$$d = \frac{cz}{H_0} - \frac{v_r}{H_0}$$

$$\sigma_d = \frac{v_r}{H_0} \rightarrow \frac{\sigma_d}{d} = \frac{v_r}{H_0 d}$$

Redshift wins over other distance indicators at large distance.

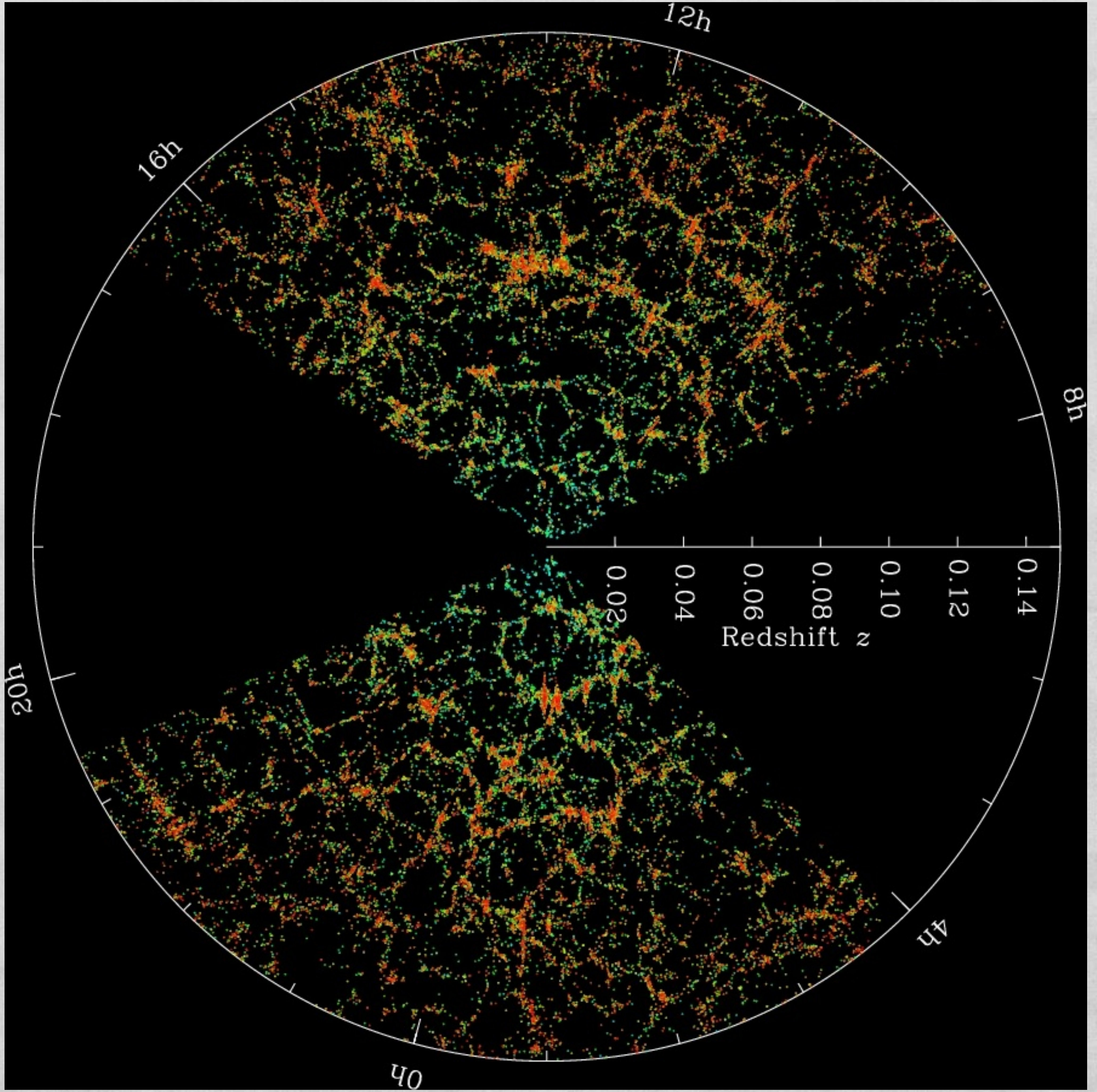


DENSITY FLUCTUATIONS

Since it is so hard to measure distances a lot of effort has been spent on measuring density fluctuations. They also have the advantage that they can tell us more than just the basic cosmological parameters. They probe the nature of dark matter and alternative theories to gravity.

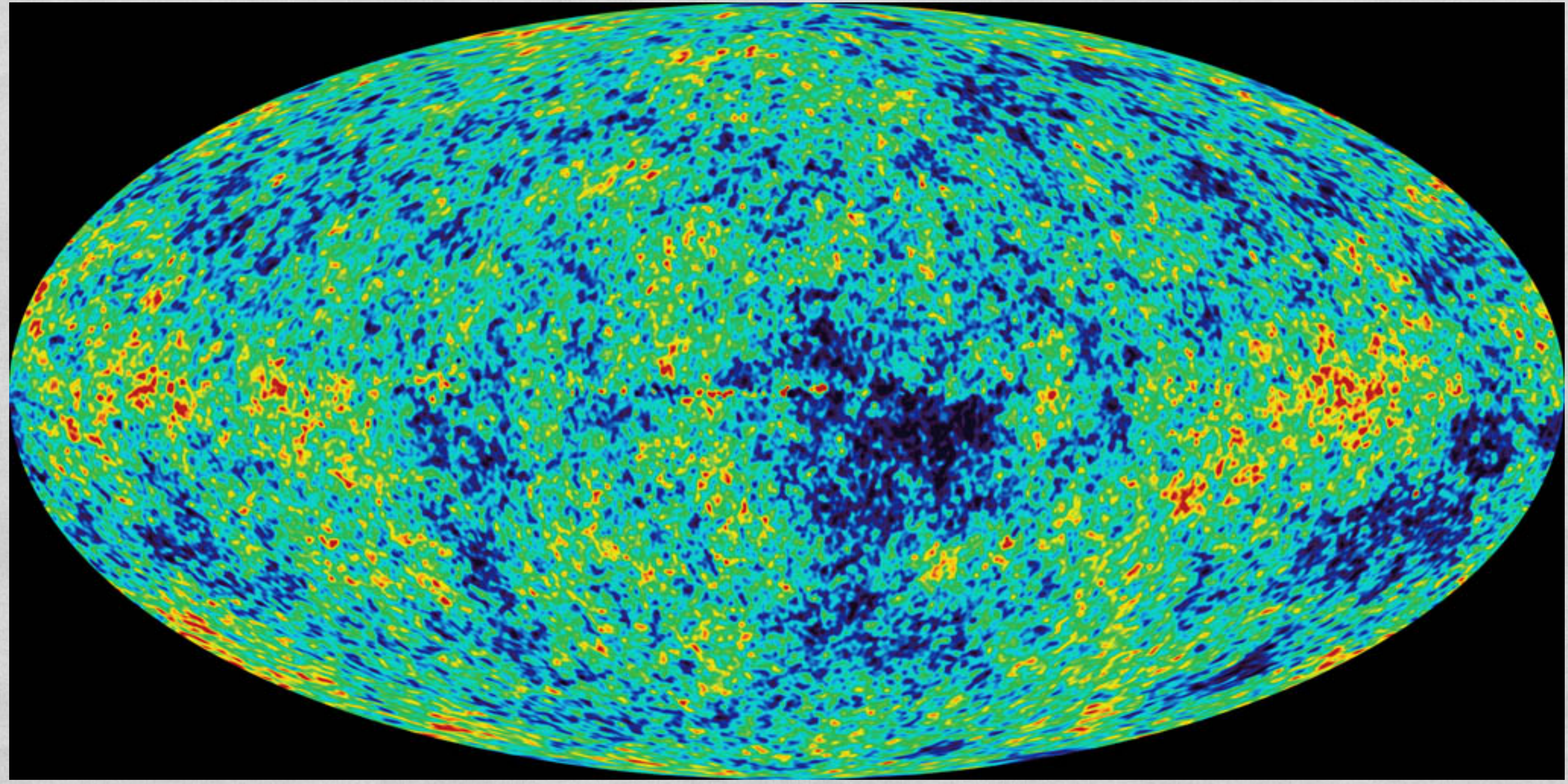
We can see density fluctuations in the galaxy distribution and those must be related to the underlying matter distribution.

The most standard way of quantifying the distribution of galaxies is from the correlation function.



DENSITY FLUCTUATIONS

Also there are fluctuations in the CMB.

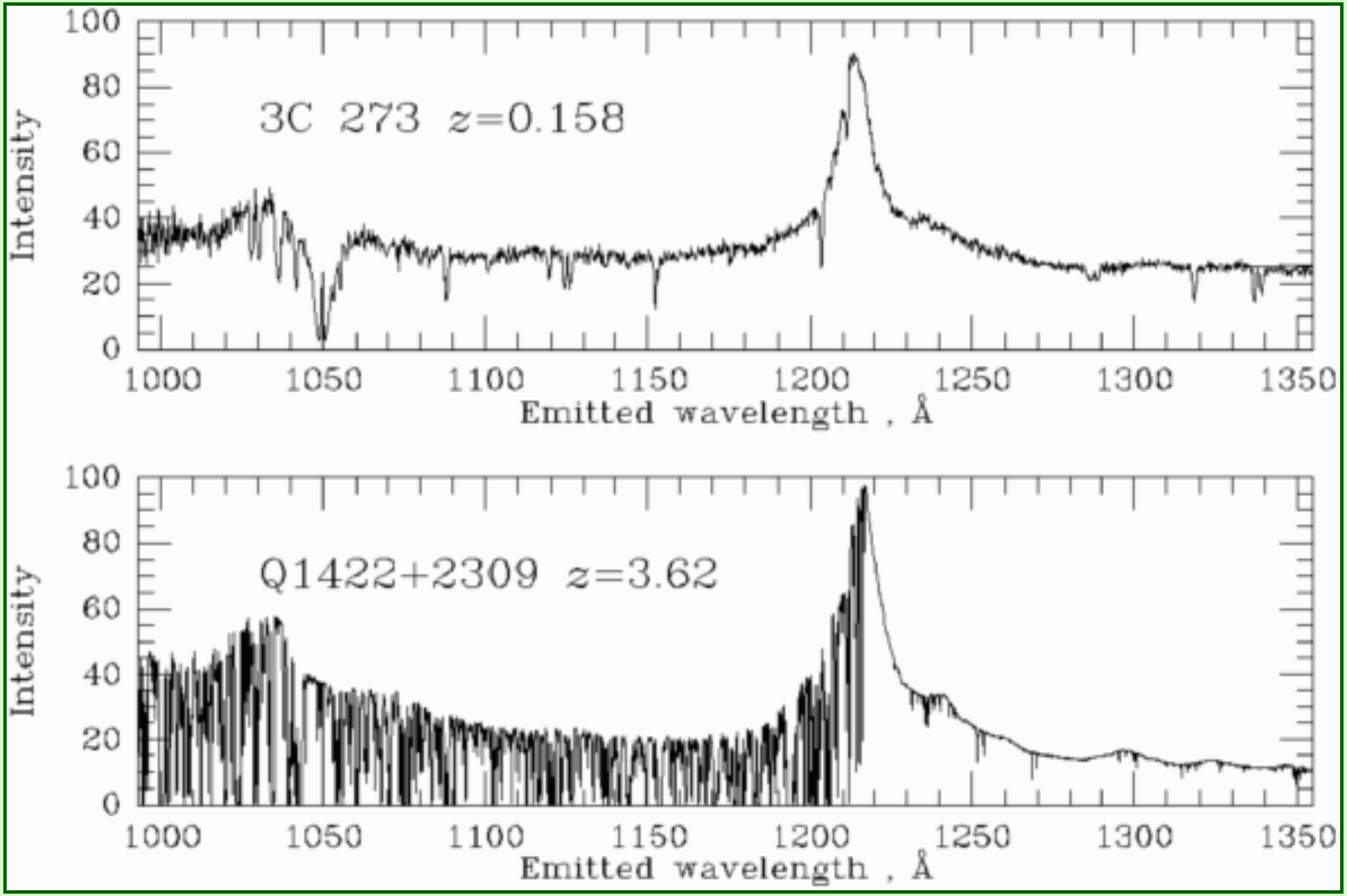


HIGHER REDSHIFT

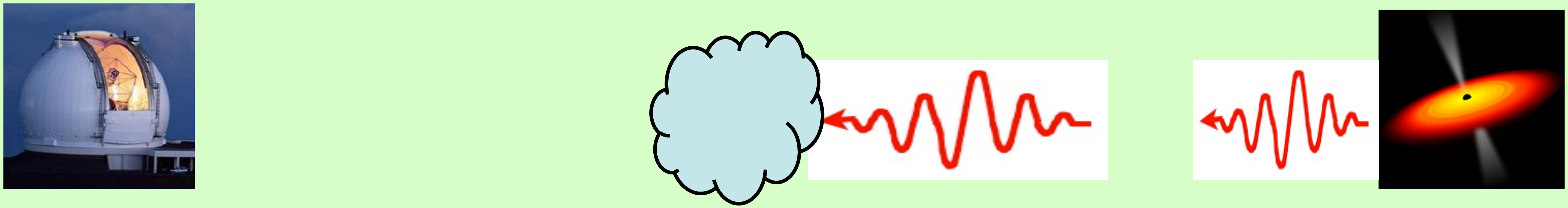
It is very challenging to measure many galaxies at higher redshift because they are fainter. There are some galaxy redshift surveys at $z \sim 1$ and even at $z \sim 3$, but the volume probed is rather small.

A more successful approach is to look at the Lyman alpha forest, which doesn't measure galaxies but still measures density fluctuations.

The Lyman alpha Forest



The Lyman alpha Forest

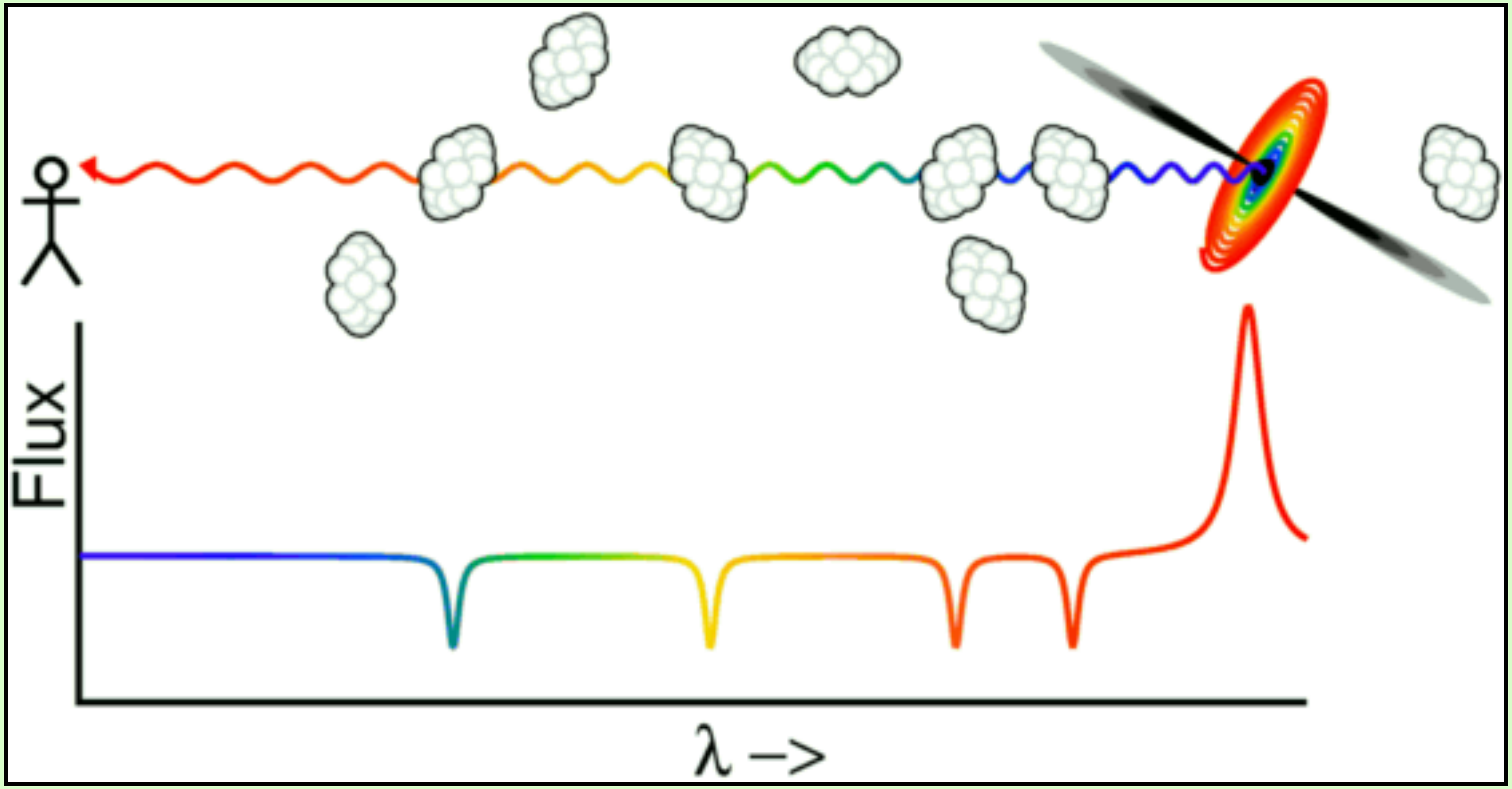


$z=0$ z_{HI} z_{QSO}

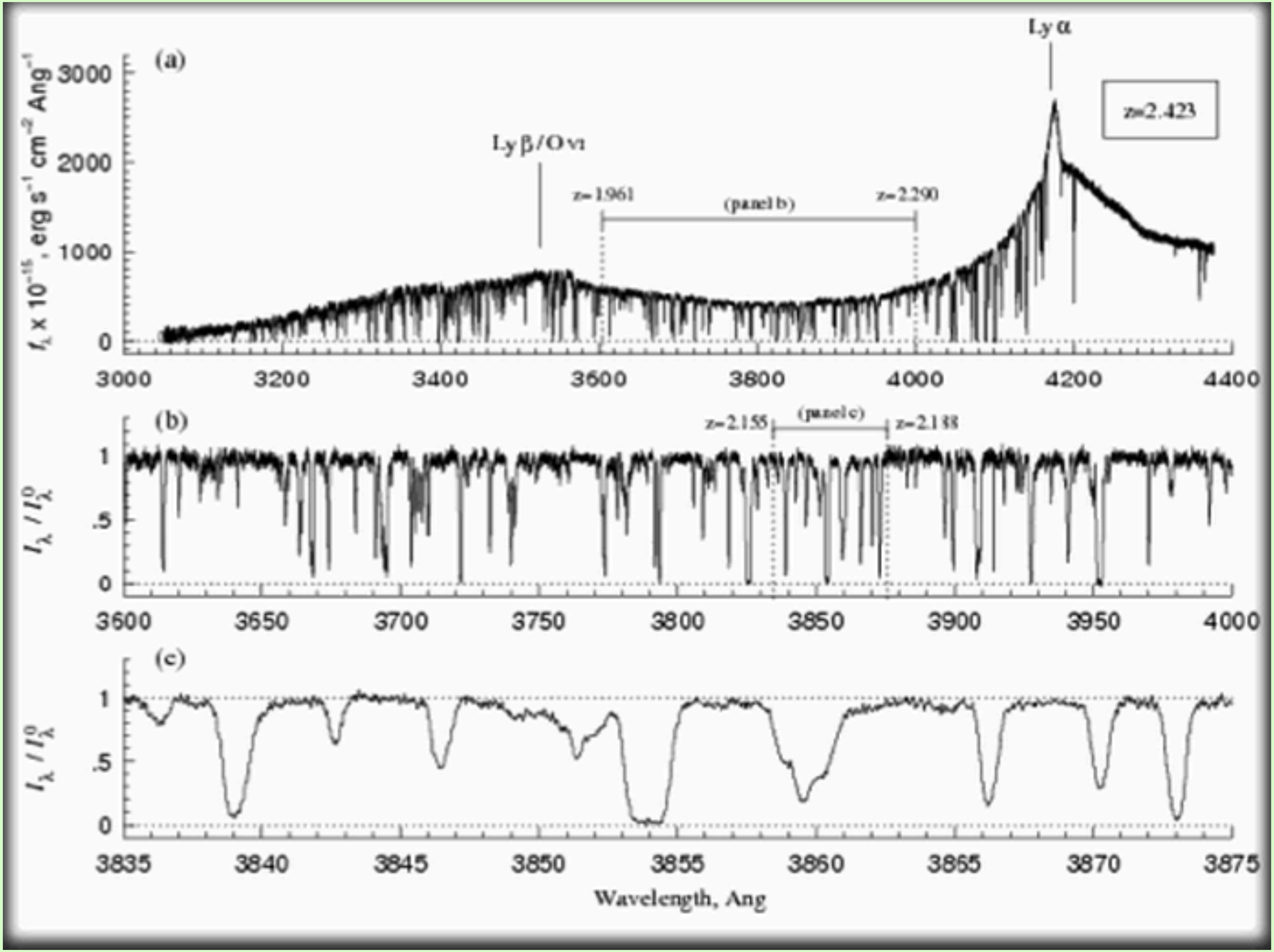
- A Lyman-α photon (1216 Å) emitted by the quasar has a longer wavelength by the time it encounters the HI cloud and so it will not be absorbed.

- The shorter wavelength photon emitted by the quasar that has stretched to 1216 Å by the time it encounters the HI cloud can be absorbed.

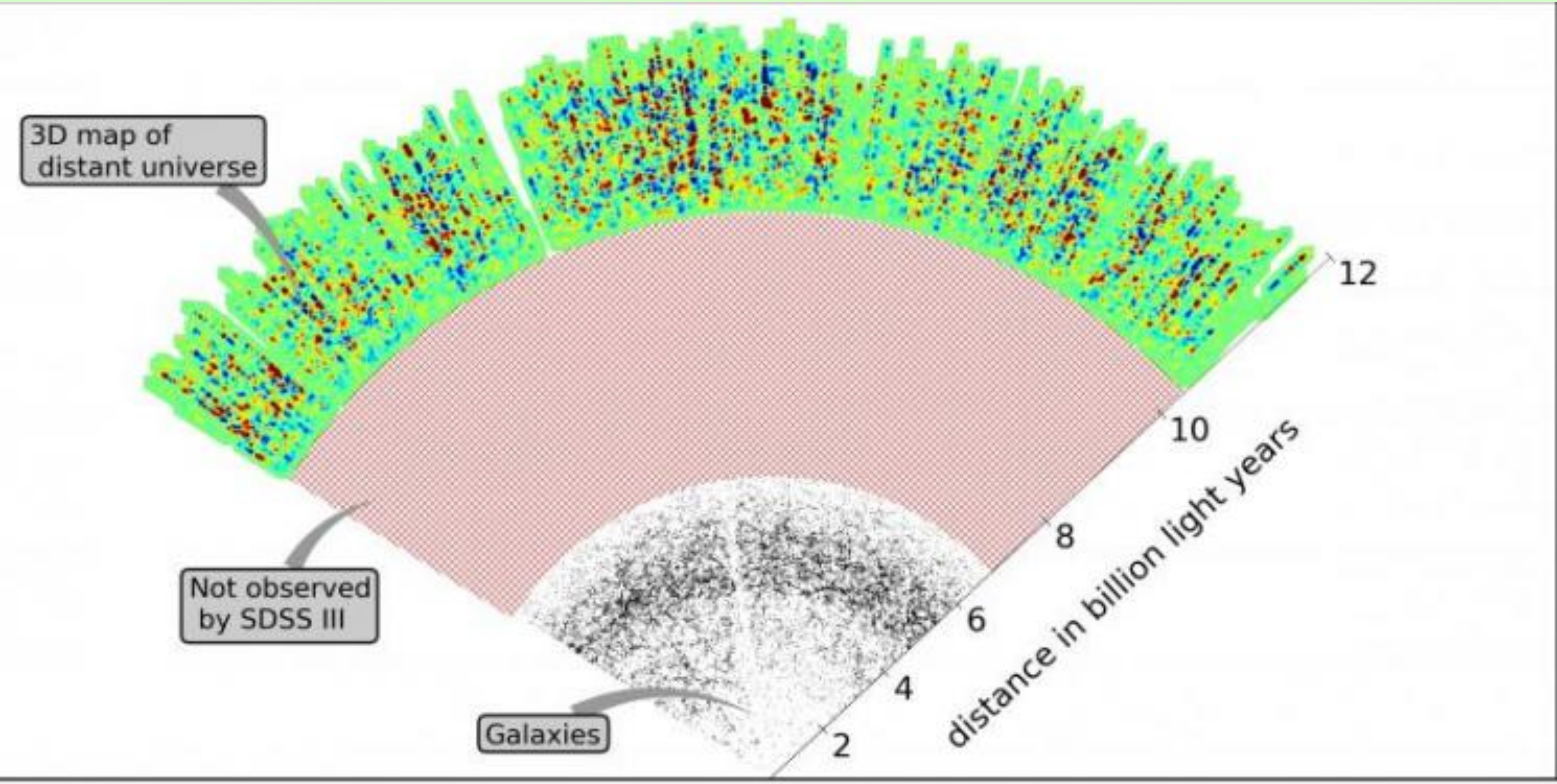
The Lyman alpha Forest



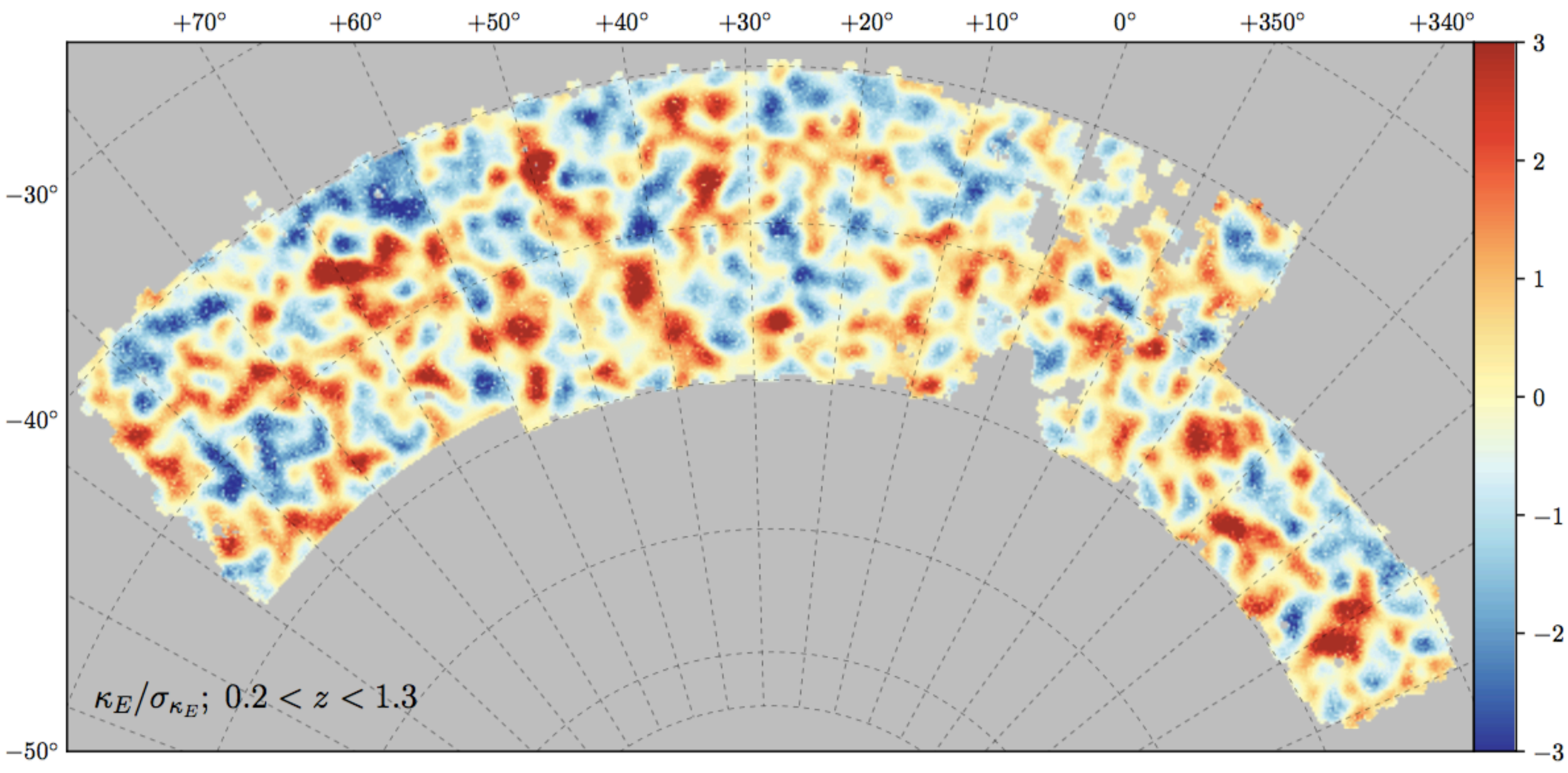
The Lyman alpha Forest



The Lyman alpha Forest



DENSITY FLUCTUATIONS FROM GRAVITATIONAL LENSING



This map shows not fluctuations in number of galaxies, but in mass as measured from weak gravitational lensing (DEES)

CORRELATION FUNCTIONS

Whether one is talking about galaxies, the Lyman alpha forest, or the fluctuations in the CMB the statistic we use to describe them is generally a correlation function.

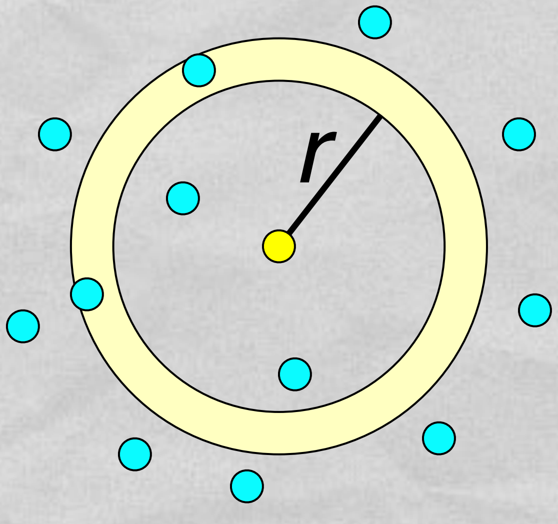
Conceptually it is much easier to picture the correlation function of points so we will focus on galaxies.

CORRELATION FUNCTIONS

The 2pt correlation function is the excess probability to find two galaxies separated by a distance r compared to random.

For a random distribution of points with density n,

$$n \cdot dV = n \cdot 4\pi r^2 dr$$



The total number density of pairs at that separation is

$$\frac{n}{2} \cdot n \cdot 4\pi r^2 dr = n^2 2\pi r^2 dr$$

CORRELATION FUNCTIONS

So the excess compared to random can be written as

$$n^2 2\pi r^2 dr[1 + \xi(r)]$$

or the correlation function ξ(r) is given by

$$\xi(r) = \frac{n_{pairs,data}(r)}{n_{pairs,rand}(r)} - 1 = \frac{DD(r)}{RR(r)} - 1$$

CORRELATION FUNCTIONS

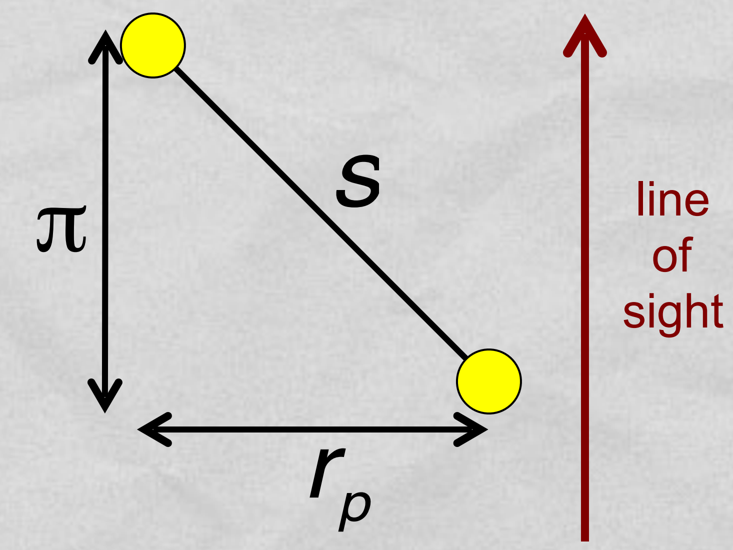
The nice thing about this is that it is rather straightforward to compute. Even if your sample has a complex geometry, you can place random points in it and then count them up.

It turns out that the simple expression is a biased estimator of the true correlation function so the Landy-Szalay estimator is often used instead.

$$\xi(r) = \frac{DD(r) - 2DR(r) + RR(r)}{RR(r)}$$

CORRELATION FUNCTIONS

You may have noticed that the correlation function depends on distance, so we run right back into the same problems we were talking about earlier.

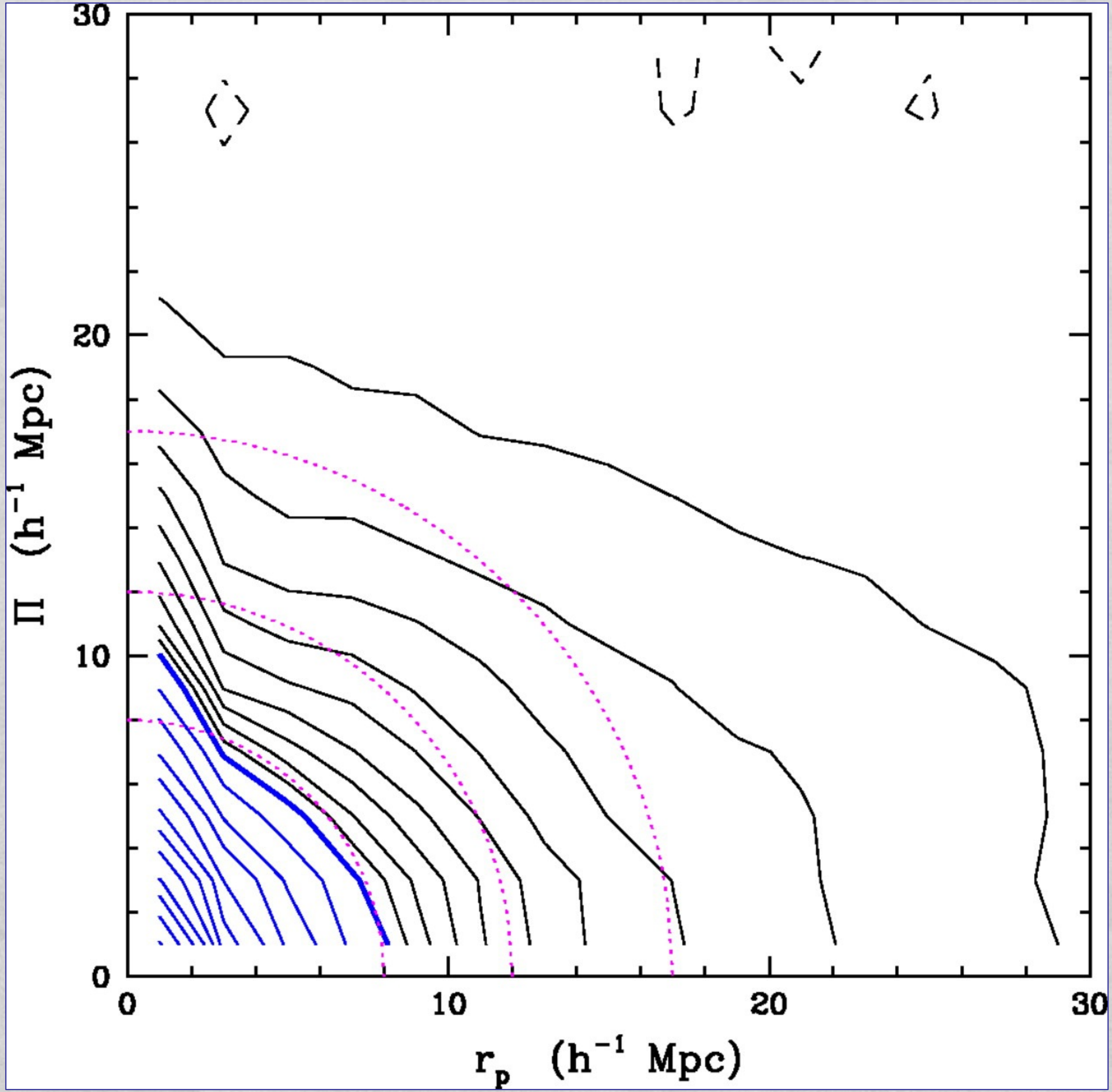


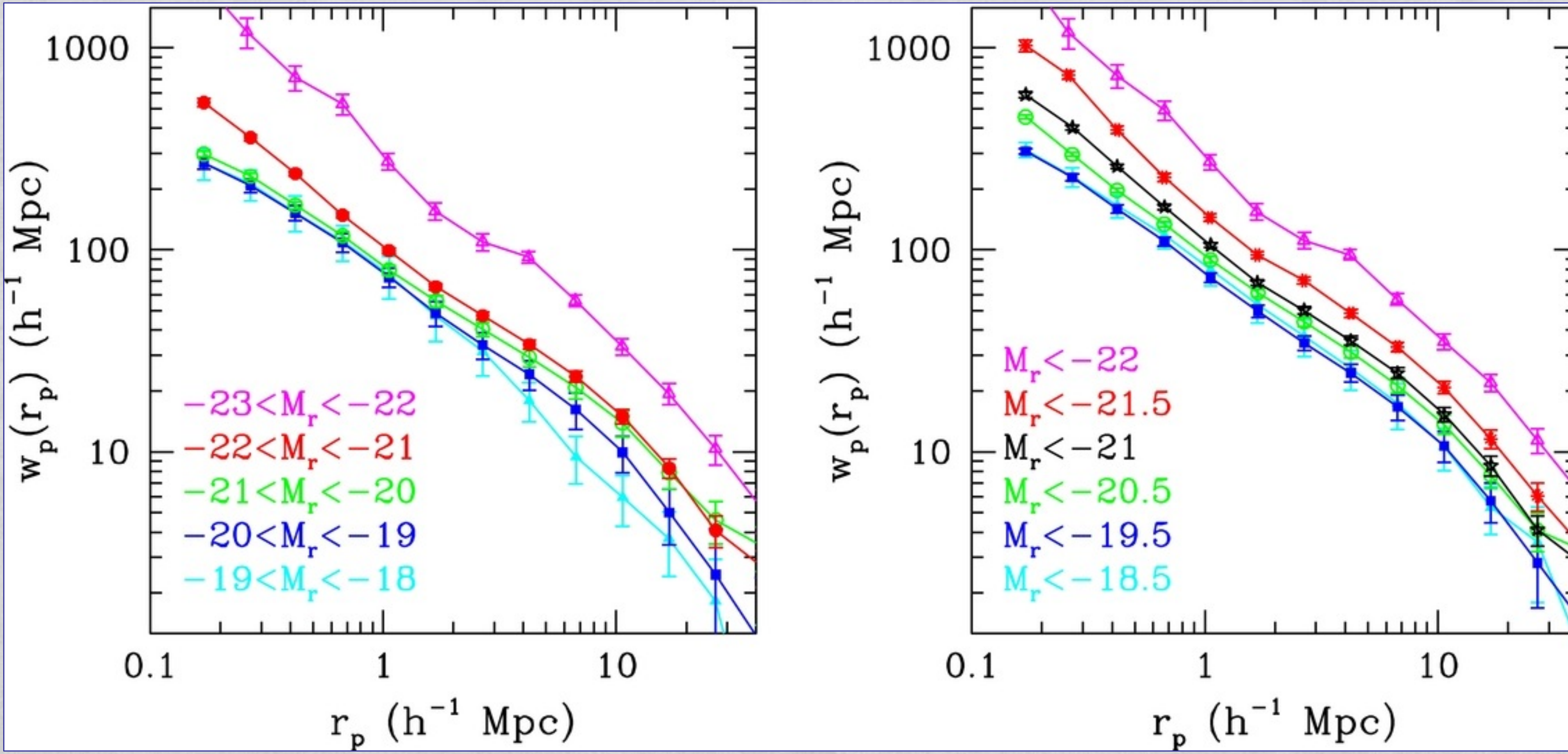
To get around this, usually the projected correlation function is measured instead of the 3D one.

$$w_p(r_p) = 2\int_0^{\pi_{max}} \xi(r_p, \pi)\, d\pi$$

The two coordinates behave differently not just because one is a velocity times the Hubble parameter, but because velocity doesn't only measure distance.

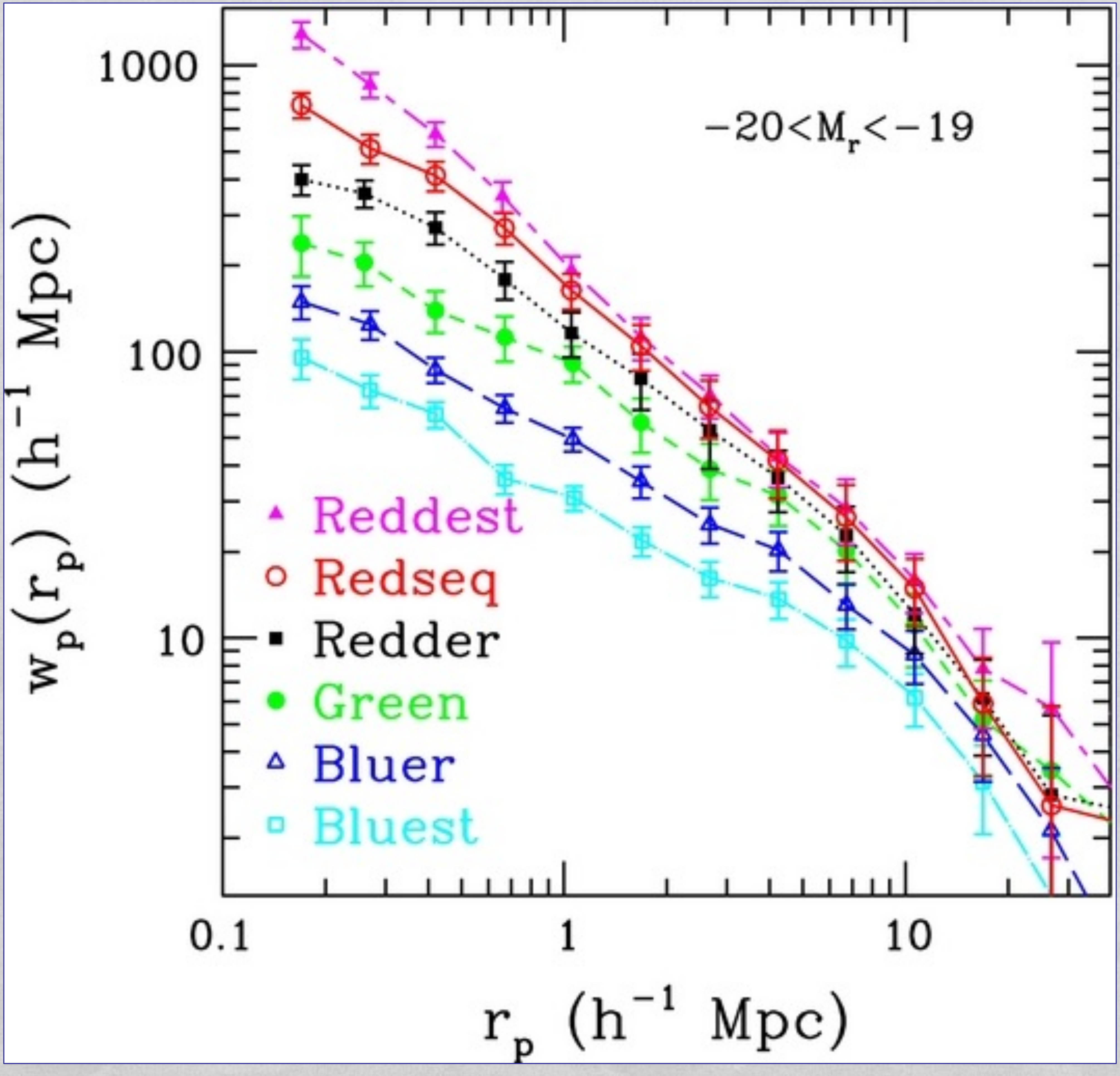
Galaxies also have peculiar velocities on top of the Hubble flow because of gravity. This causes what is called redshift distortions that can also be used to constrain cosmology because those velocities are the result of density fluctuations.





We can still get nice measurements of the correlation function. When we look at the correlation function binned by galaxy luminosity or color we see that the amplitude changes.

This brings up an important point, it makes sense that in regions of more matter there will be more galaxies, but how many more galaxies?



As we can see from these figures, different types of galaxies seem to trace the matter distribution differently.

Galaxies are a *biased* tracer of the mass distribution with different types of galaxies have different biases.

GALAXY BIAS

This adds a complication, studying the galaxy distribution is not really studying the mass distribution. We can describe the relation with a bias parameter b,

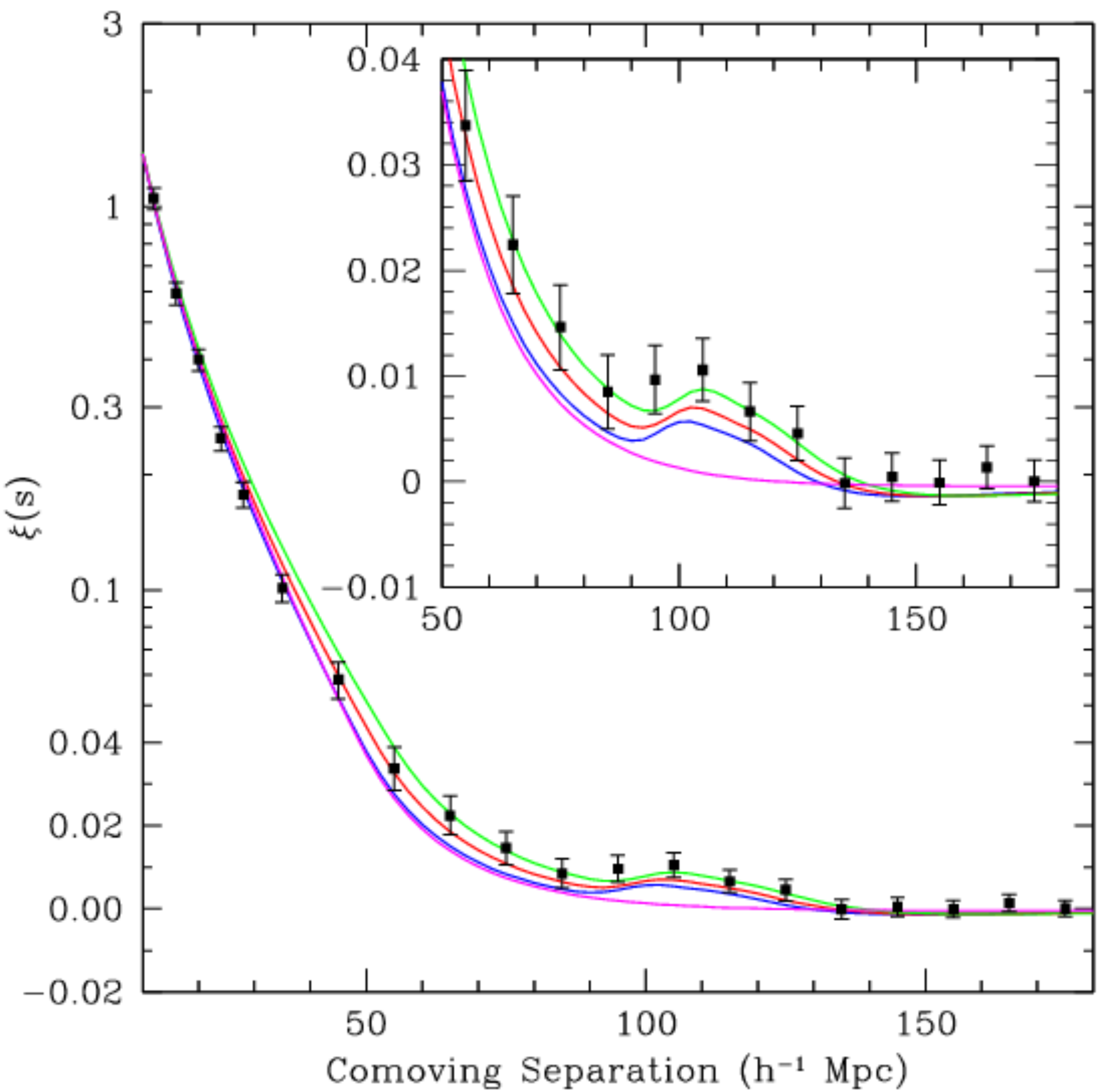
$$b(r) = \sqrt{\frac{\xi_{galaxy}(r)}{\xi_{mass}(r)}}$$

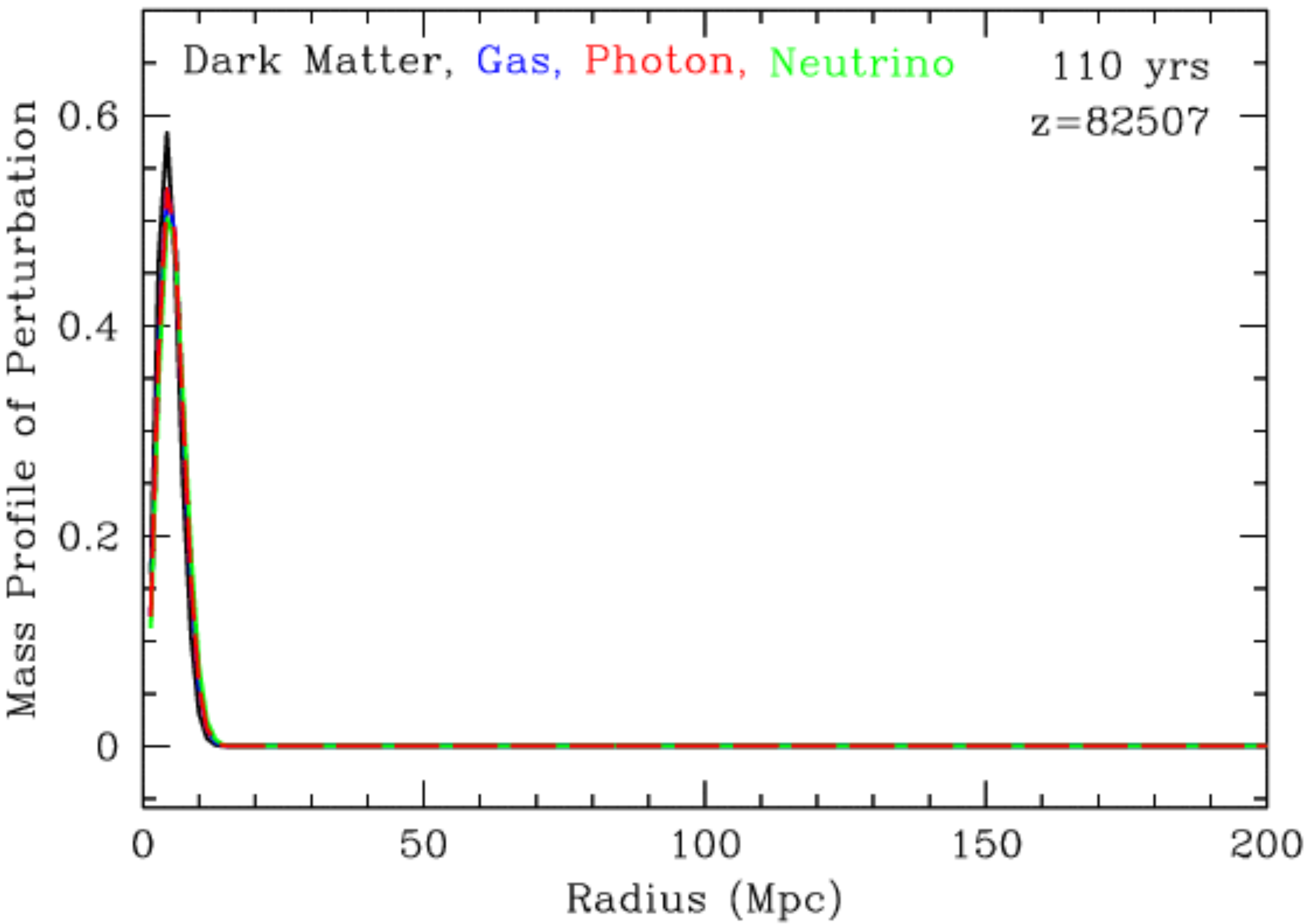
It is thought that on large scales b(r) should become constant, so then you are studying the power spectrum with one extra unknown. On small scales the bias should encode all the complexities of galaxy formation.

BARYON ACOUSTIC OSCILLATIONS

One enormous breakthrough in this has been the discovery of baryon acoustic oscillations (BAO) in the galaxy correlation function. Remember the first peak in the CMB, caused by sound waves. Well that peak in the matter distribution should have grown in time and still be visible today. That is what the BAO is, a slight overdensity of galaxies at the scale of the sound horizon in the CMB.

This enhanced power is particularly useful because it doesn't depend on bias, since it is a length scale. Instead we have found a standard ruler whose height depends on how densities grow over time.




Dark Matter, Gas, Photon, Neutrino
110 yrs
z=82507
Mass Profile of Perturbation
0.6
0.4
0.2
0
0
50
100
150
200
Radius (Mpc)

BAO AT DIFFERENT Z

Currently one of the goals in cosmology is to measure the baryon acoustic peak at various redshifts.

This is a standard ruler so it can be used to determine a(t) without assumptions about the nature of cosmology.

Furthermore the amplitude of the peak measures the growth of structure which can be used as a test of GR separate from cosmology

The reason this is difficult is that the length scale is large and thus requires large surveys (WFIRST).

COSMIC MICROWAVE BACKGROUND

ADVANTAGES OF THE CMB

The cosmic microwave background offers a number of advantages when used for estimating cosmological parameters.

* Perturbations are small ($\sim 10^{-5}$) so linear perturbation theory can be used.

* Physics is straightforward, just a photon-baryon fluid and gravity. Especially compared to galaxies with star formation, supernova feedback, black holes, shocks, cosmic rays, magnetic fields, etc.

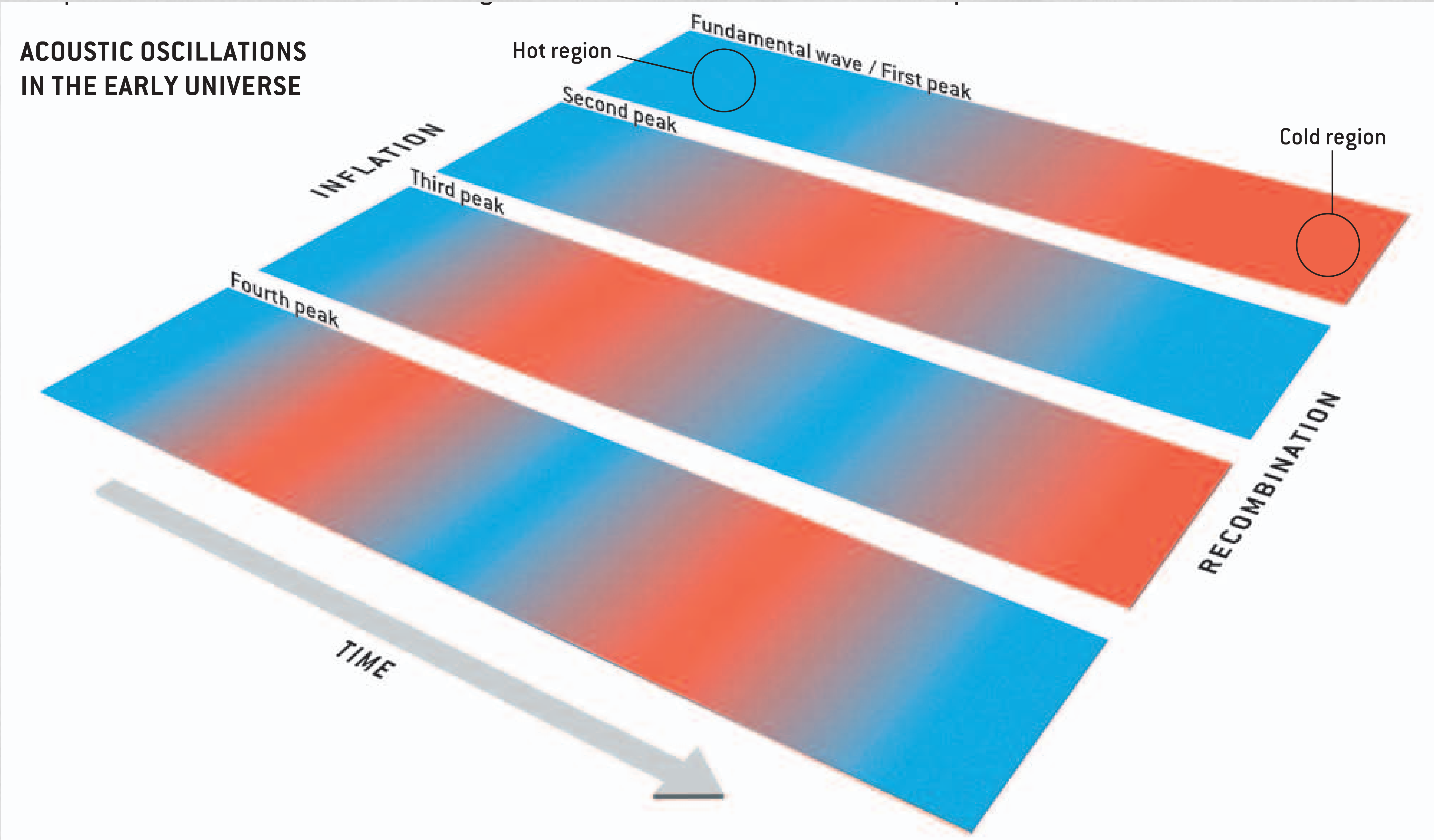
BASIC PHYSICS

The CMB is a photon-baryon fluid. Density fluctuations will travel at the sound speed. These waves compress the fluid causing density and temperature fluctuations.

We measure temperature fluctuations, not total density fluctuations and thus are sensitive to the phase of the waves. Waves that are reaching their maximum or minimum amplitude will create the largest temperature fluctuations.

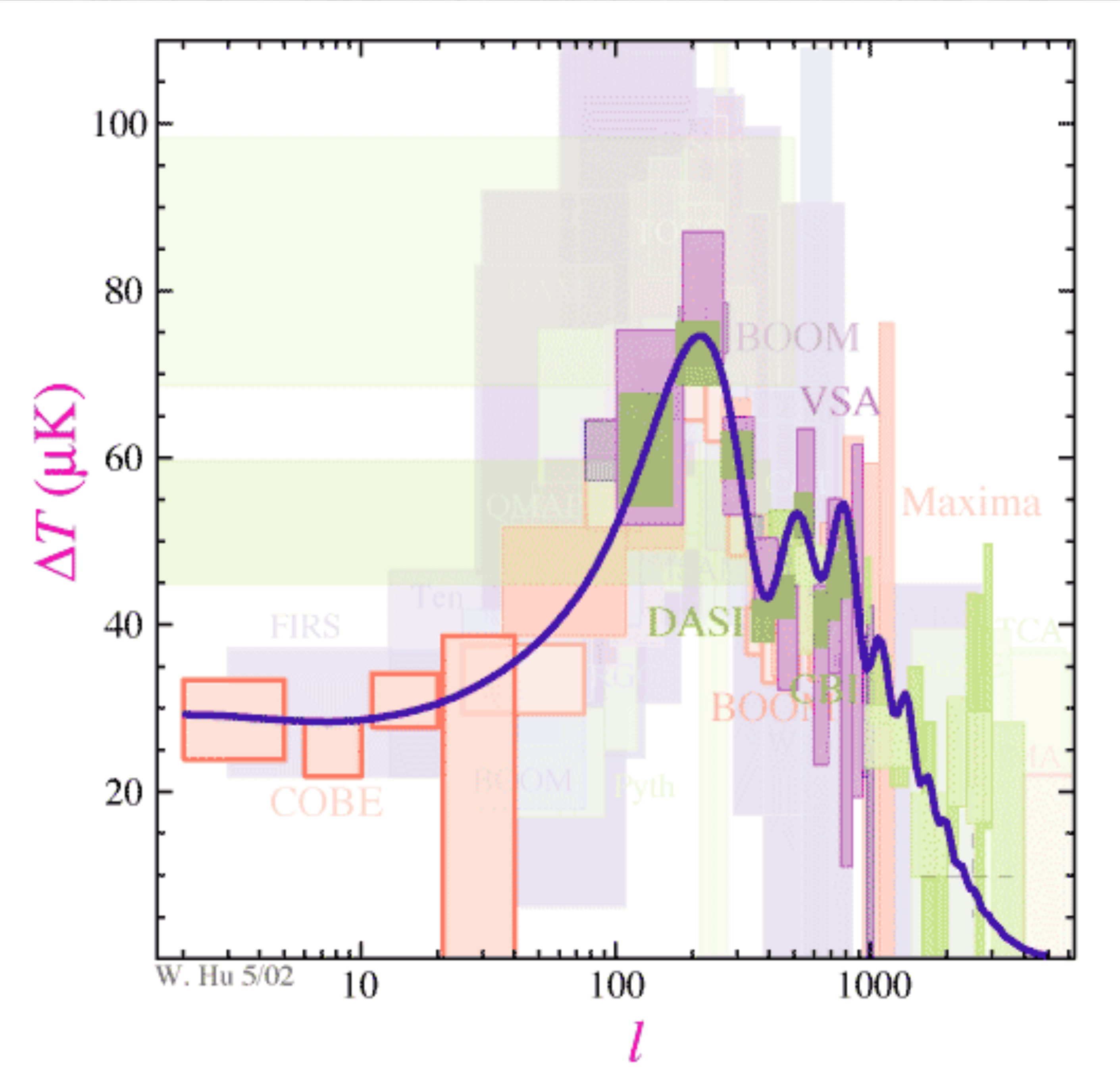
All waves start off in the same phase after inflation. Regions with density maxima cause waves to form and propagate through the photon-baryon fluid. At the time of recombination all the waves will have traveled a distance the sound horizon=$c_s t_r$, where t_r is the age of the Universe at recombination.

If that distance is exactly half a wavelength then that wave will be at a minima at recombination, if that distance is one wavelength then it will be a maxima. So we expect to see oscillations in the power spectrum corresponding to maxima at the sound horizon divided by integers and divided by half integers.


ACOUSTIC OSCILLATIONS
IN THE EARLY UNIVERSE
Hot region
Fundamental wave / First peak
Second peak
Third peak
Fourth peak
Cold region
INFLATION
RECOMBINATION
TIME

THE CMB POWER SPECTRUM

The CMB power spectrum is usually expanded in spherical harmonics. Thus different spatial scales correspond to a different Y_{lm}. The y-axis often changes so beware of that. Note that the dipole is always first removed before measuring the power spectrum. The dipole is believed to be a doppler shift do to our motion and not a feature of the CMB.



THE FIRST PEAK

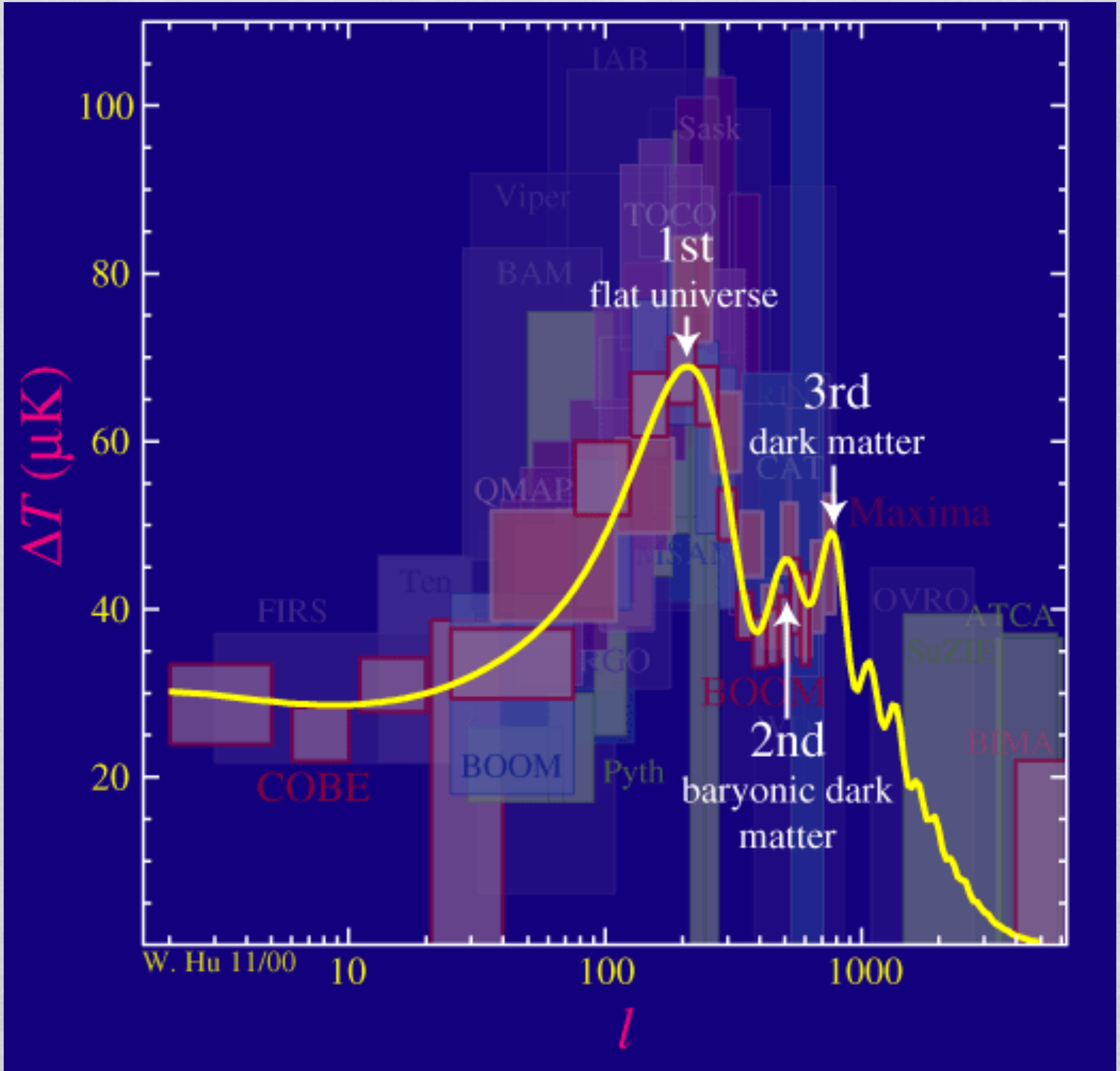
As mentioned before the location of the first peak is therefore a standard ruler and can be used to primarily measure the curvature of the Universe. If one is more interested in Ω_m and Ω_Λ then it measures the sum of the two. The location is slightly shifted by other parameters that end up changing the time of recombination, but since that time is t_r~400,000yr this shift is not large.

SECOND PEAK

We have ignored gravity so far in our discussion. Baryons increase the mass of our fluid. The greater the baryon fraction the more the fluid responds to being compressed. If there were no baryons the photons would just oscillate in the potential, compression or rarefaction would be the same. The more baryons there are the more compression and rarefaction differ (because of gravity), the larger a difference in amplitude we expect to see between the first and second peak. The relative amplitude of these two peaks primarily measure the baryon fraction.

THIRD PEAK

We need to consider not only the gravity of baryons but also of dark matter. Similar to baryons, dark matter increases the fluctuations, but since it is not oscillating in the fluid it enhances the total amplitude of the wave, not just the compression part. So dark matter increases all three peaks. The contribution of dark matter and baryons can't be disentangled in the first peak. But the relative amplitude of the 3rd peak compared to the second and first breaks this degeneracy.



ΔT (μK)
l
100
80
60
40
20
10
100
1000
W. Hu 11/00
1st
flat universe
3rd
dark matter
2nd
baryonic dark
matter
IAB
Sask
Viper
TOCO
BAM
QMAP
Maxima
CAT
FIRS
Ten
OVRO
ATCA
SuZIE
COBE
BOOM
Pyth
RGO
BIMA

SILK DAMPING

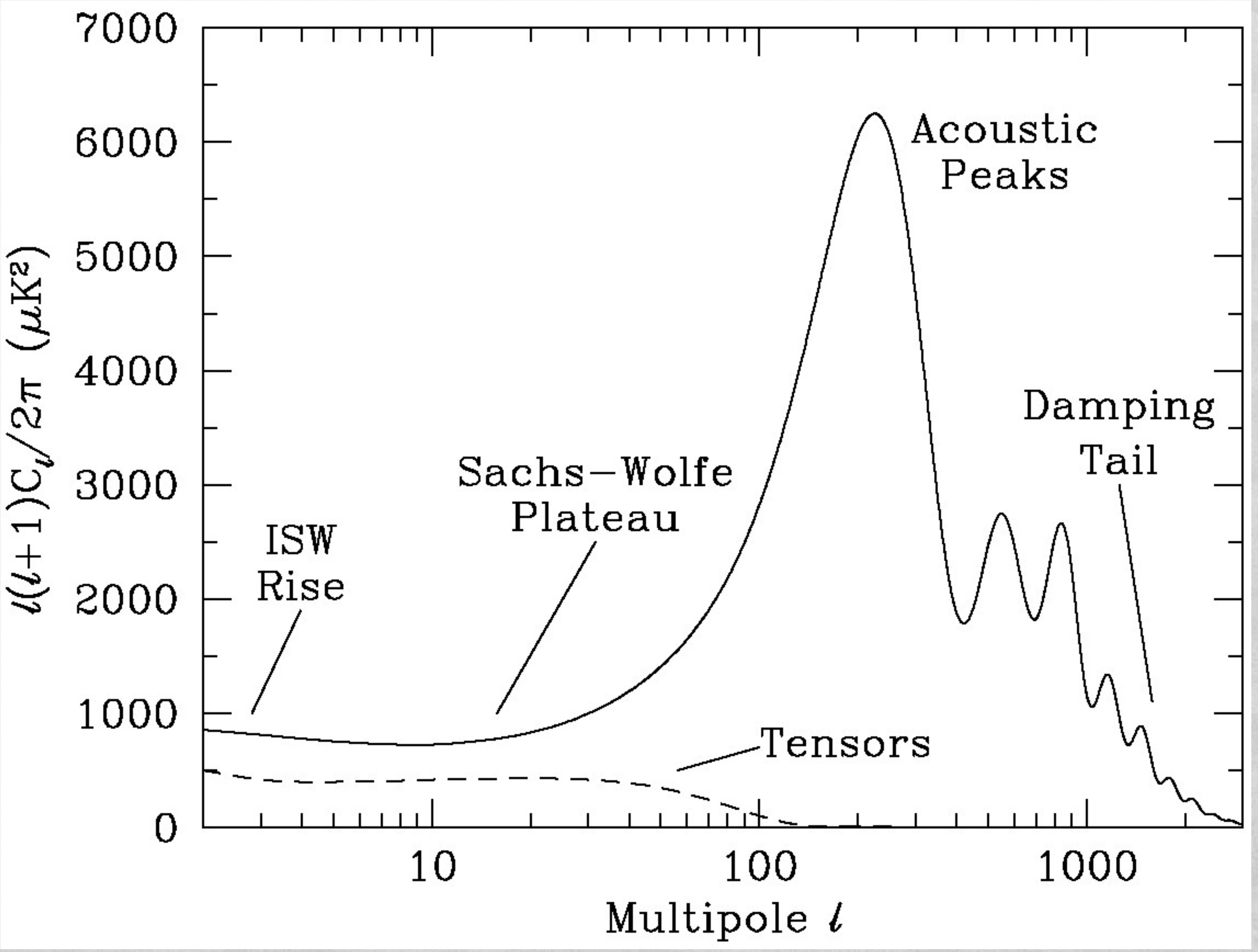
Silk (or diffusion) damping is why the remaining peaks get smaller and smaller. Decoupling doesn't happen instantaneously but takes some time. During that time, photons move around. On scales close to the mean free path of the photons temperature differences are washed out. The details depend on the baryon and dark matter density and thus provide a constancy check on results from the first three peaks.

SACHS-WOLFE

Another effect that has to be considered is that at the time of photon decoupling the last scattering surface doesn't all sample the same potential. Some photons will be in slightly over dense or under dense regions. These photons must expand more or less energy leaving their potential well and anisotropies are introduced. This is mostly important on the largest scales. For a flat Universe:

$$\frac{\delta T}{T} = \frac{\phi}{3c^2}$$

Primary CMB Anisotropies



SECONDARY EFFECTS

The CMB power spectrum is divided into primary and secondary effects. The primary effects are the ones that occurred in the photon-baryon fluid and during decoupling. Secondary effects are any that occur after the photons decouple. The main secondary effects are from reionization, gravitational lensing, the integrated Sachs-Wolfe effect and the Sunyeav Zeldovich effect.

REIONIZATION

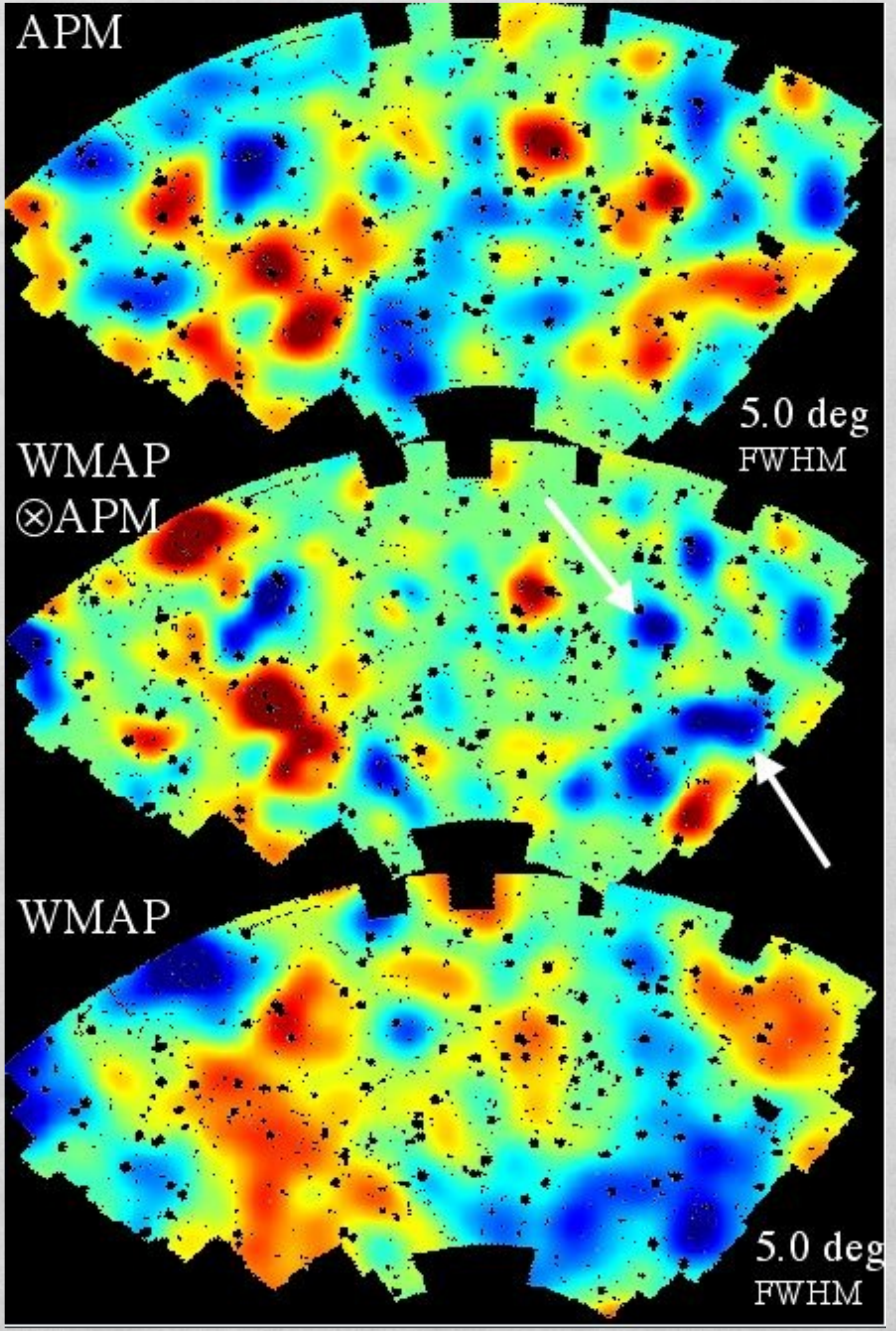
We usually say that after decoupling the photons and neutral atoms never interact again, but that isn't exactly true. Eventually stars and quasars form and they emit UV light which re-ionizes the hydrogen and helium in the Universe. Then there are free electrons and CMB photons will Thompson scatter off of them. This introduces polarization and a slight bump at large scales. The bump depends on when reionization occurs and thus this parameter has to be added to models to correctly fit the data. Current measurements place reionization between $z = 7$ and 20.

INTEGRATED SACHS-WOLFE

The Sachs-Wolfe effect is the temperature variations induced on the surface of last scattering by fluctuations in the potential wells that those photons found themselves in. However, as the photons traverse the Universe to us they encounter other potential wells on the way. The added effect of all these if they are linear (small) is called the integrated Sachs-Wolfe effect.

One might expect there to be no effect from other potential wells. Photons are blueshifted as they enter the potential and redshifted as they exist for no net effect. This is true if the potential doesn't evolve, but gravity should cause the potential well to become deeper as the photon traverses it, thus the photon leaves with less energy than it entered with. This effect is most important on large scales. Note this is an independent test of gravity which controls the growth rate of perturbations.

This effect can also be studied by correlating the CMB with observed structure in the Universe. Unlike other CMB measurements here we are not just looker at the power in the CMB, but the actual location of colder spots.

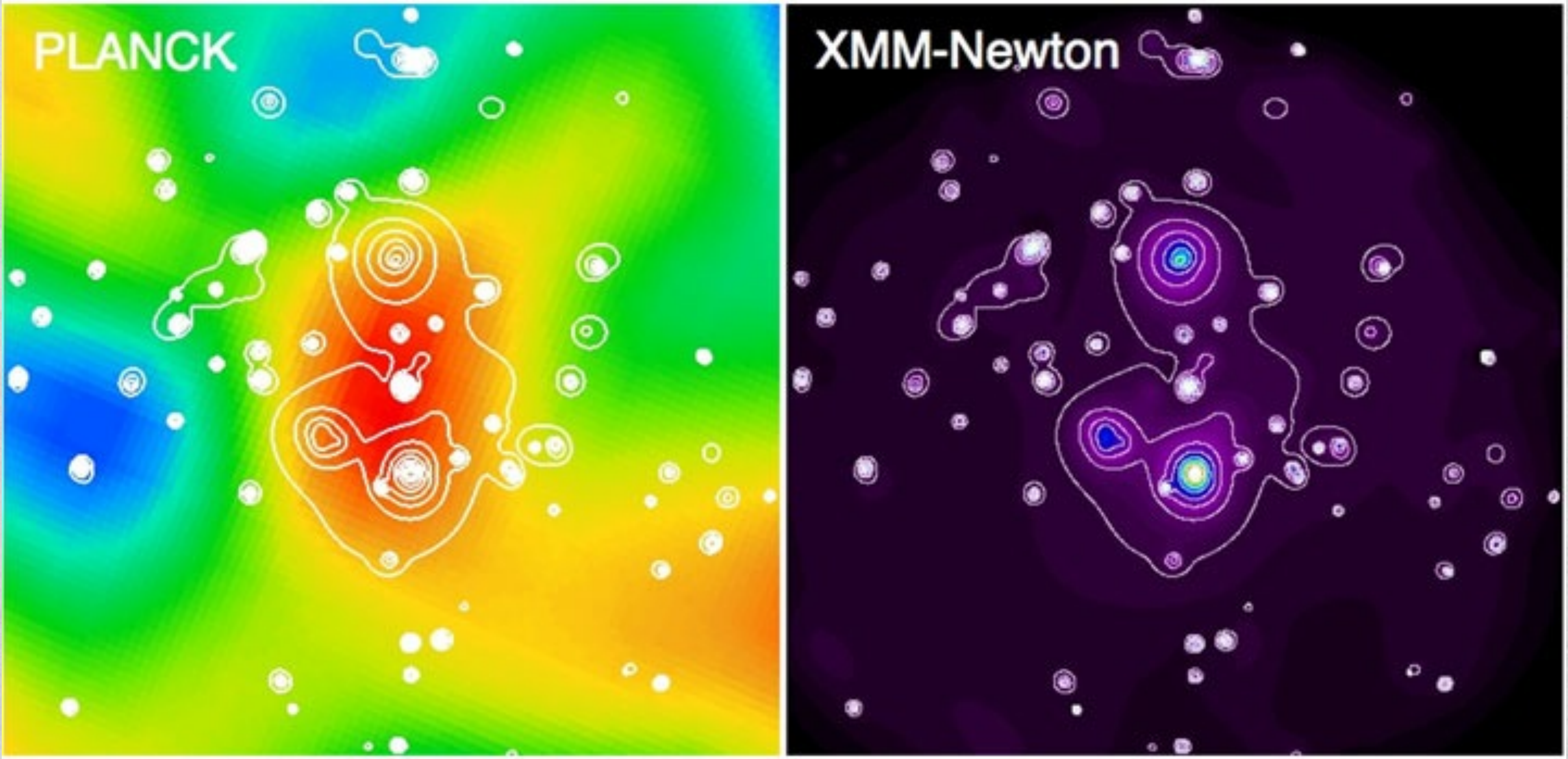


REES-SCIAMA

If CMB photons encounter a nonlinear structure that evolves on the time scale that it takes them to transverse it this is called the Rees-Sciama effect. The time it takes for photons to cross a structure is $t \sim 1000\text{kpc}/3\times10^5$ kpc/Gyr ~3.25Myr/Mpc. If the structure is undergoing very quick evolution its potential can change during this time thus shifting the photons energy. Most structures do not evolve this quickly in which case there is no effect. This effect is only important on small scales.

SUNYAEV-ZELDOVICH

The SZ effect is the Compton scattering of CMB photons off of hot gas primarily in clusters. The thermal SZ refers to the case where the motion of the hot gas electrons is thermal. In this case the effect is simply a reduction of the apparent temperature of the CMB photons as they remain a black body. The kinetic SZ is when the gas has bulk motion, then the black body curve is changed. SZ effects small angular scales, but is also an opportunity to find clusters and in a distance independent method.



Images taken of the same cluster with Planck and an X-ray telescope. The structure is picked up in Planck because of the SZ effect, the CMB photons are scattered out giving a cold spot where the cluster is. X-ray flux falls as distance squared, but absorption doesn't, making SZ a good way to find clusters at high redshift.

TENSOR PERTURBATIONS

The CMB can also be used to measure tensor perturbations. This is now the main goal in CMB experiments.

Tensor perturbations can be seen in the polarization of the CMB photons. This is a much more challenging detection. So far polarization has been detected in the CMB, but not the signal from tensor (gravitational wave) perturbations.

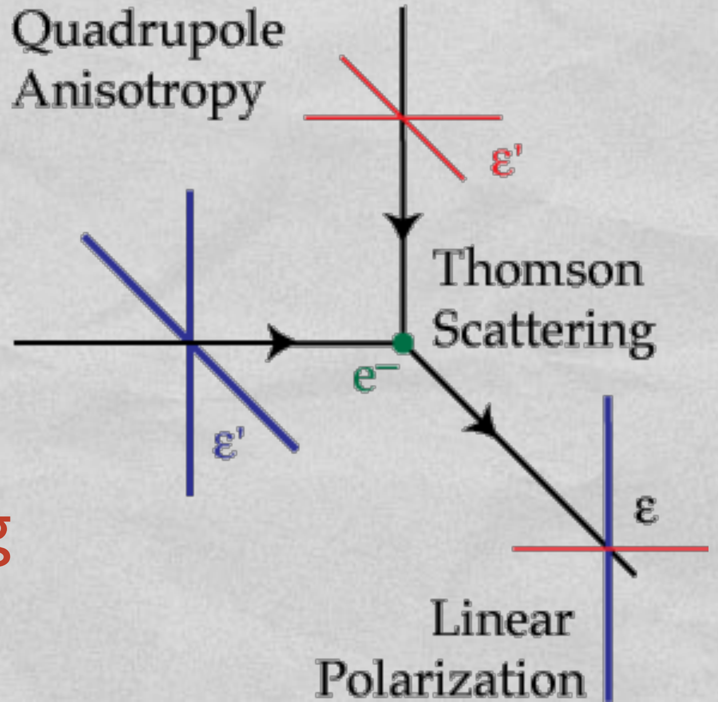
POLARIZATION OF CMB

The Thomson scattering cross-section depends on polarization.

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4\pi mc^2}\right)^2 |\hat{\epsilon} \cdot \hat{\epsilon}'|^2$$

If the incident radiation was isotropic then the scattered radiation would remain unpolarized.

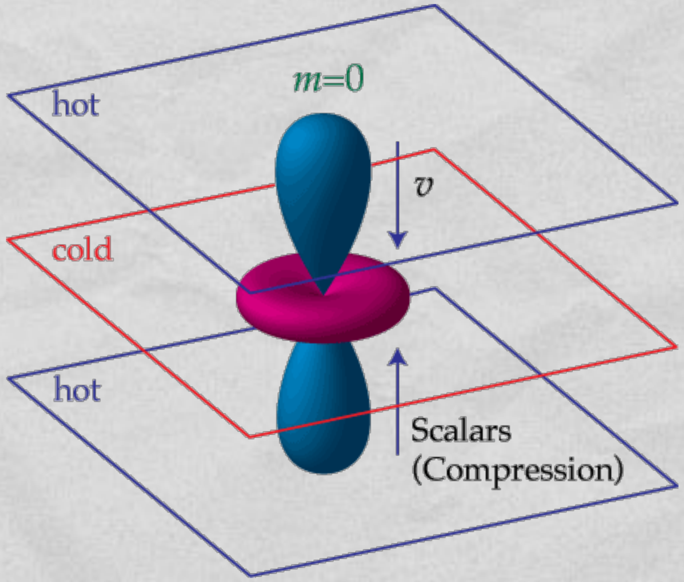
If the incident radiation field possesses a quadrupolar variation in intensity or temperature If the incoming radiation the result is linear polarization of the scattered radiation.

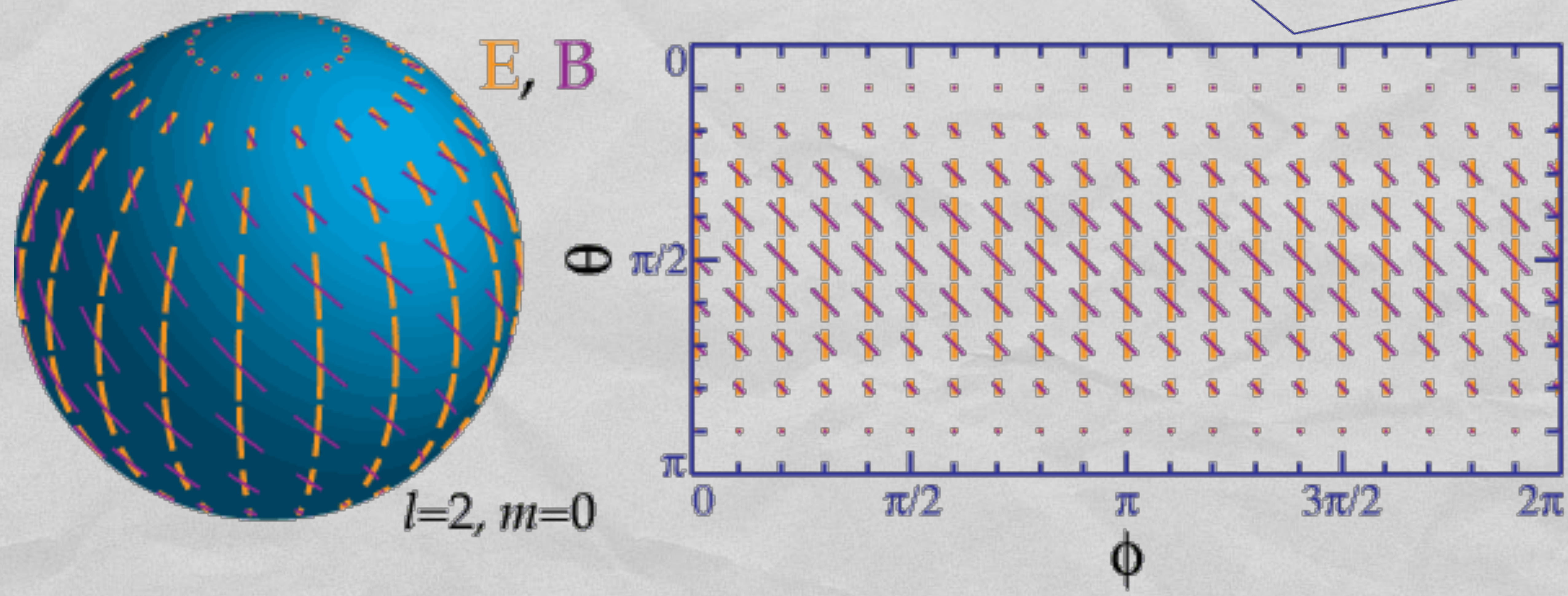


The polarization can be quantified using the U and V Stokes parameters or what is commonly done in this field by calling them E and B polarization like the electromagnetic field.

SCALAR PERTUBATIONS

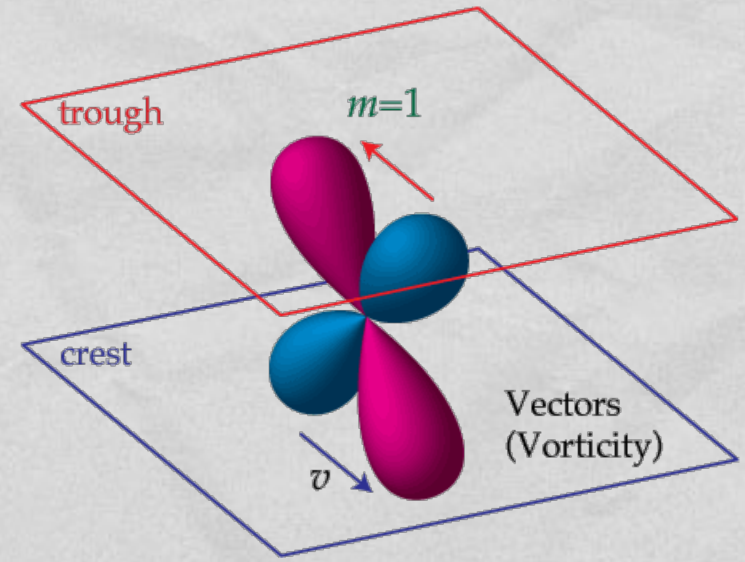
A plane wave feeling scalar perturbations would create a polarization pattern like that seen below.

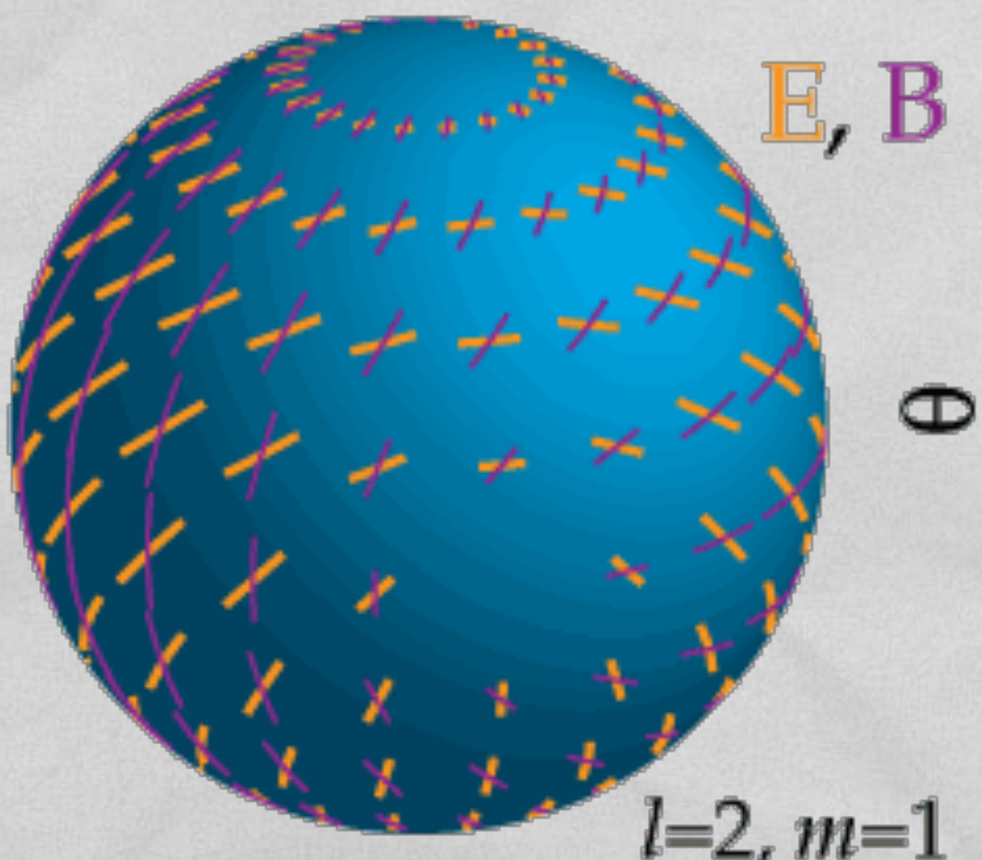


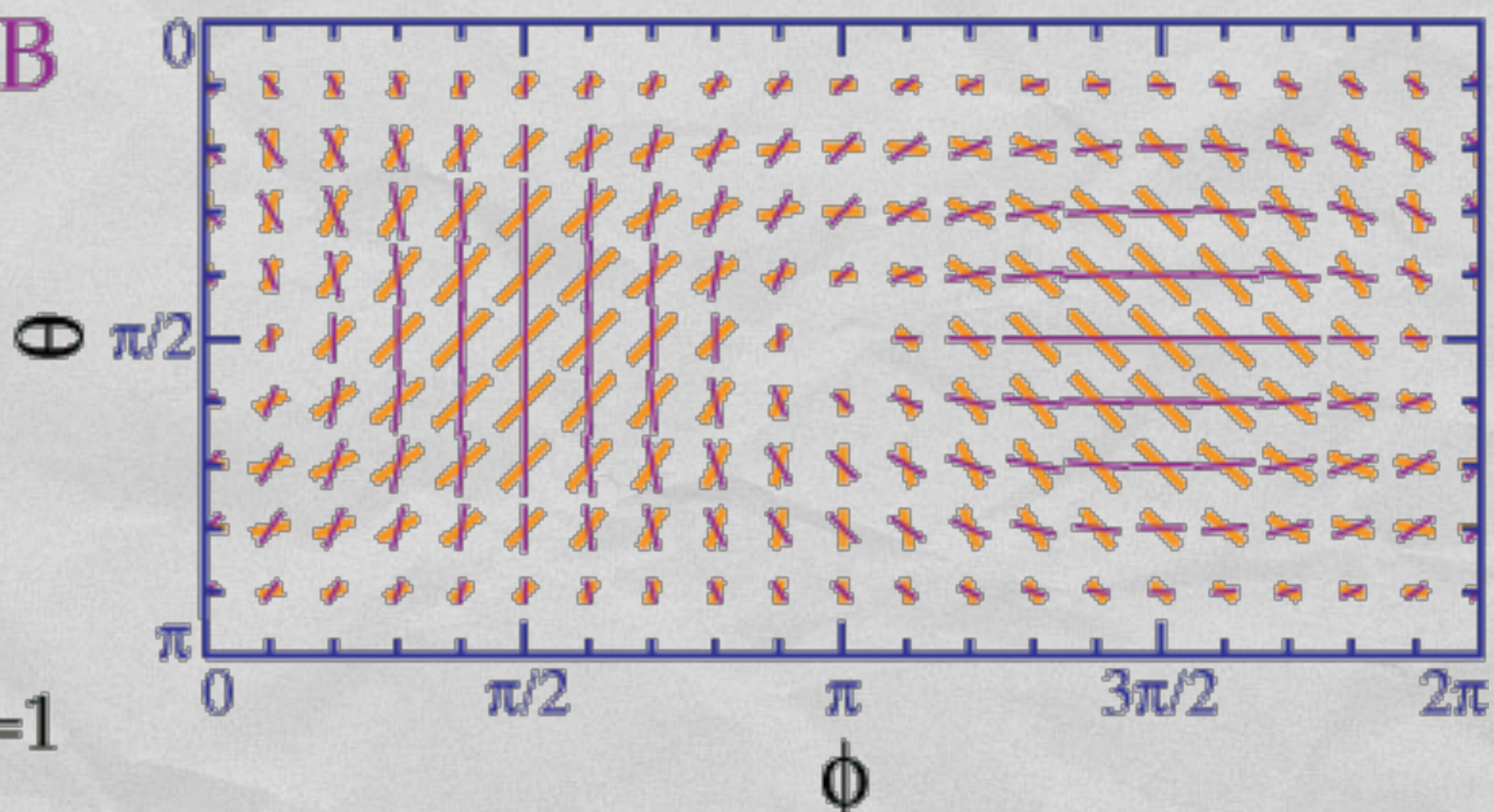


VECTOR PERTURBATIONS

A plane wave feeling vector perturbations would create a polarization pattern like this.

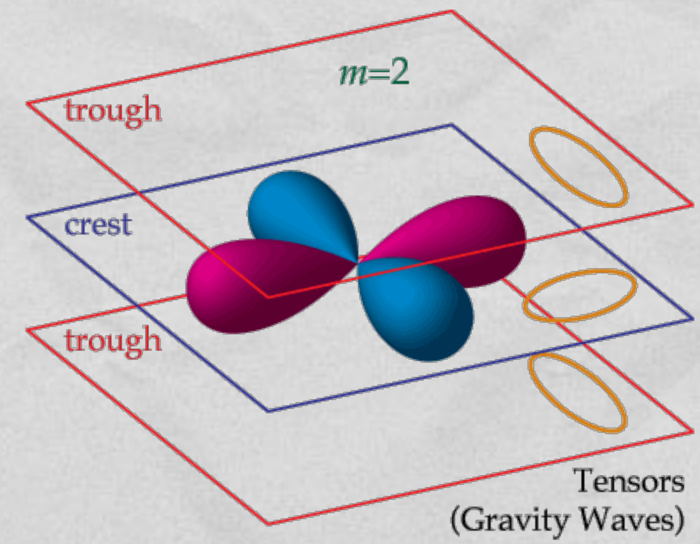


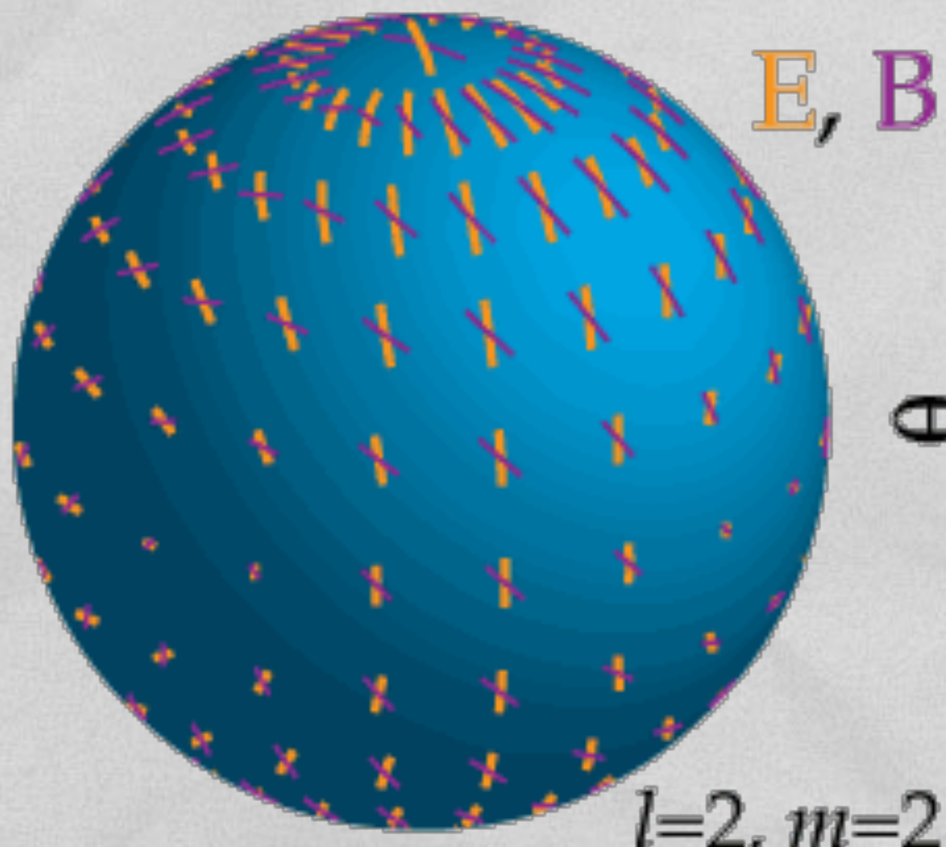


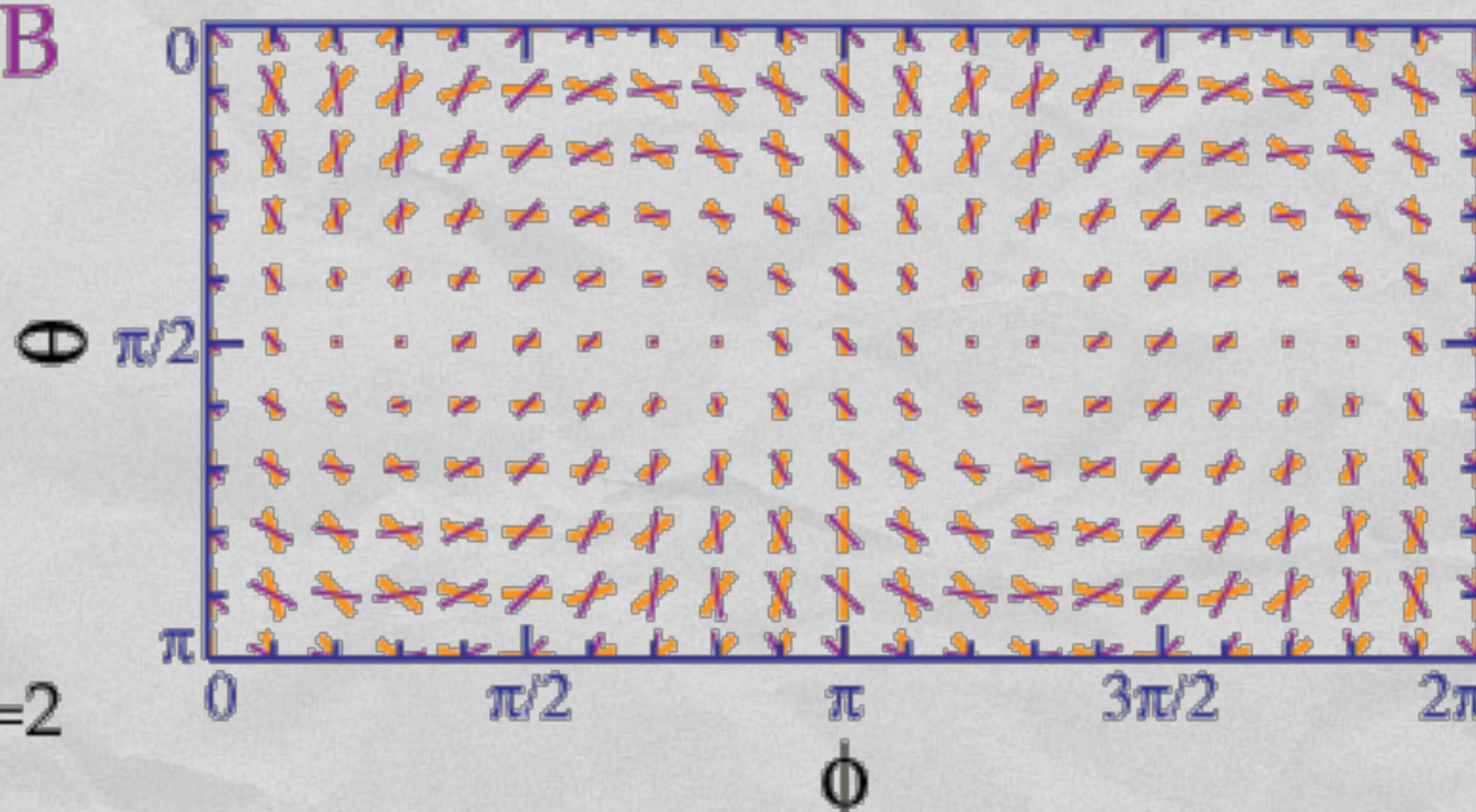


TENSOR PERTURBATIONS

A plane wave feeling tensor perturbations would create a polarization pattern like this.

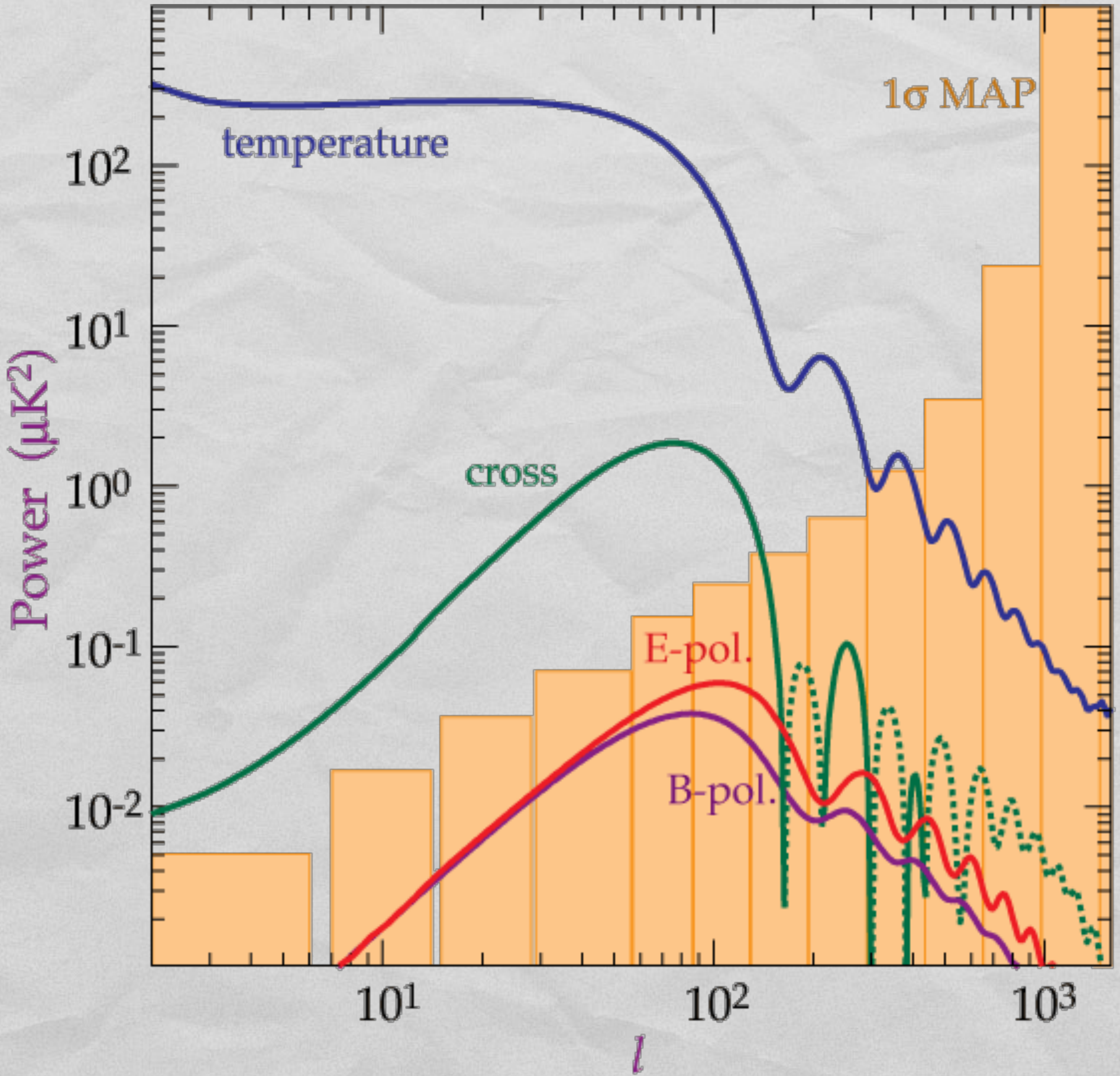




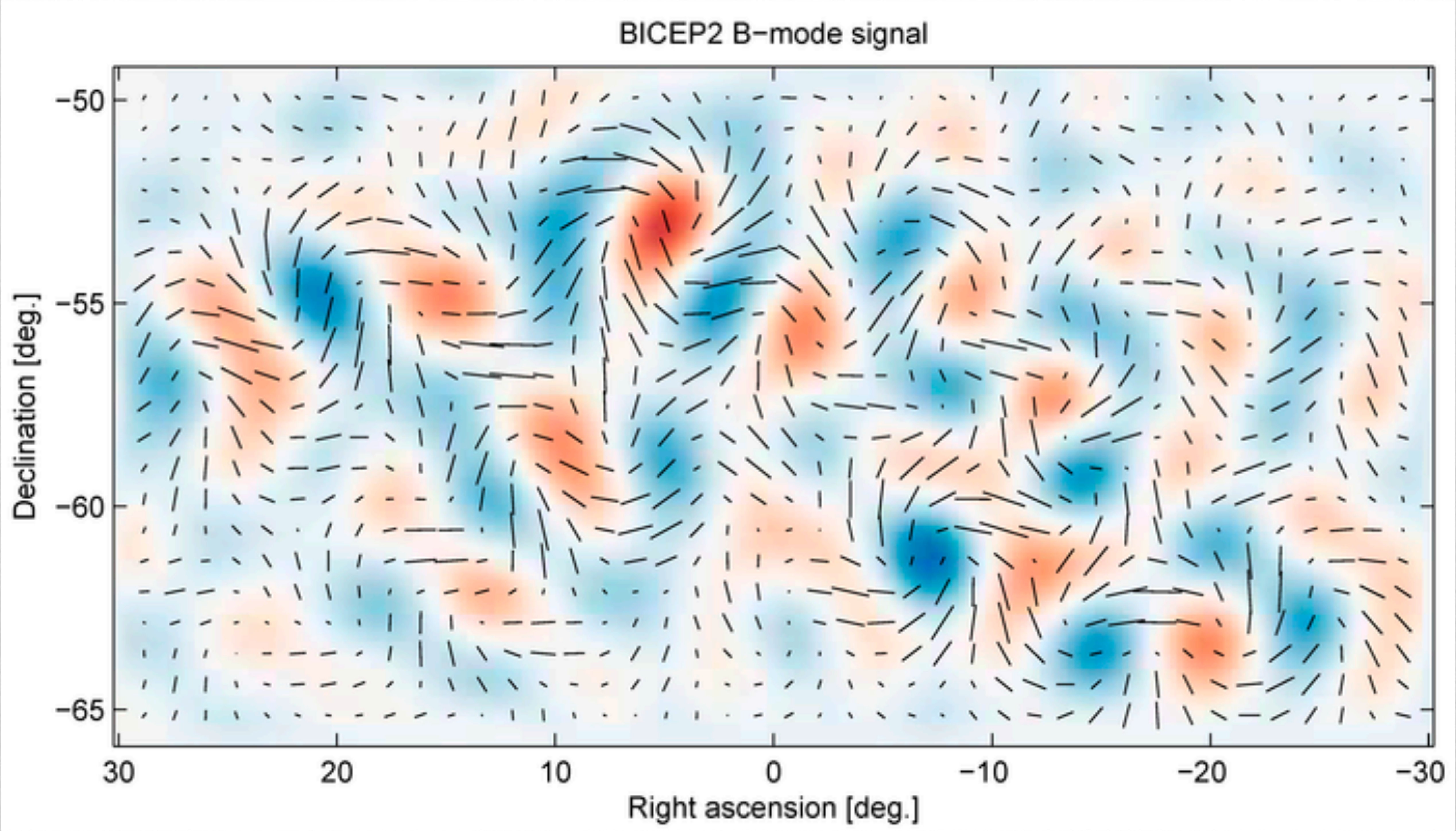


While the CMB isn't a single plane wave, the correlations between temperature fluctuations and polarization persists.

Studying the cross-correlation between temperature fluctuations, E-polarization and B-polarization make the discovery of tensor modes possible.



1σ MAP
temperature
cross
E-pol.
B-pol.
Power (μK²)
10²
10¹
10⁰
10⁻¹
10⁻²
10¹
10²
10³
l



The BICEP2 experiment claimed detection of the B-mode signal, but later analysis from PLANCK showed that this was foreground contamination. Foregrounds are a real problem because dust emission can also be polarized. Detecting this signal is the next major discovery in CMB science.

GZK LIMIT

The Greisen-Zatsepin-Kuzmin (GZK) limit is an upper limit to cosmic rays (high energy particles) in the Universe. The limit is based on that fact that very high energy particles will scatter off CMB photons. The scattering cross section increases as you approach the pion rest mass, 200 MeV. This creates an exponential cutoff to cosmic rays at around 6×10^{19}eV.

COSMOLOGY WITH CLUSTERS

Clusters can be a very strong probe of cosmology. They are essentially the hight mass end of the halo mass function which one can predict from simulations. However, they are also sensitive to some none standard parameters. If the cluster mass function is larger or smaller than you might expect in your cosmology that can be evidence that fluctuations are non gaussian, (you are probing the tail of the distribution). Or that gravity is non standard.

GRAVITATIONAL LENSING

GRAVITATIONAL LENSING

Gravitational lensing not only effects the CMB but has become a more and more important part of cosmology and astrophysics in general. Gravitational lensing has the great advantage that it directly measures projected mass, with close to no assumptions. Thus gravitational lensing of the CMB tells us about the total amount of mass fluctuations along the photons path. And gravitational lensing can be used to study the masses of other objects like clusters or galaxies and their substructure, the power spectrum of mass fluctuations, and discover things like black holes and planets.

BASIC IDEAS

In most cases of interest we can assume that the deflection happens near the lens and that the deflection is small enough that we can integrate the effects of gravity over the unperturbed path. In these cases we can project the mass into a two dimensional sheet. The mass sheet then has a surface density give by,

$$\Sigma(\vec{\xi}) = \int \rho(\vec{\xi}, z) dz$$

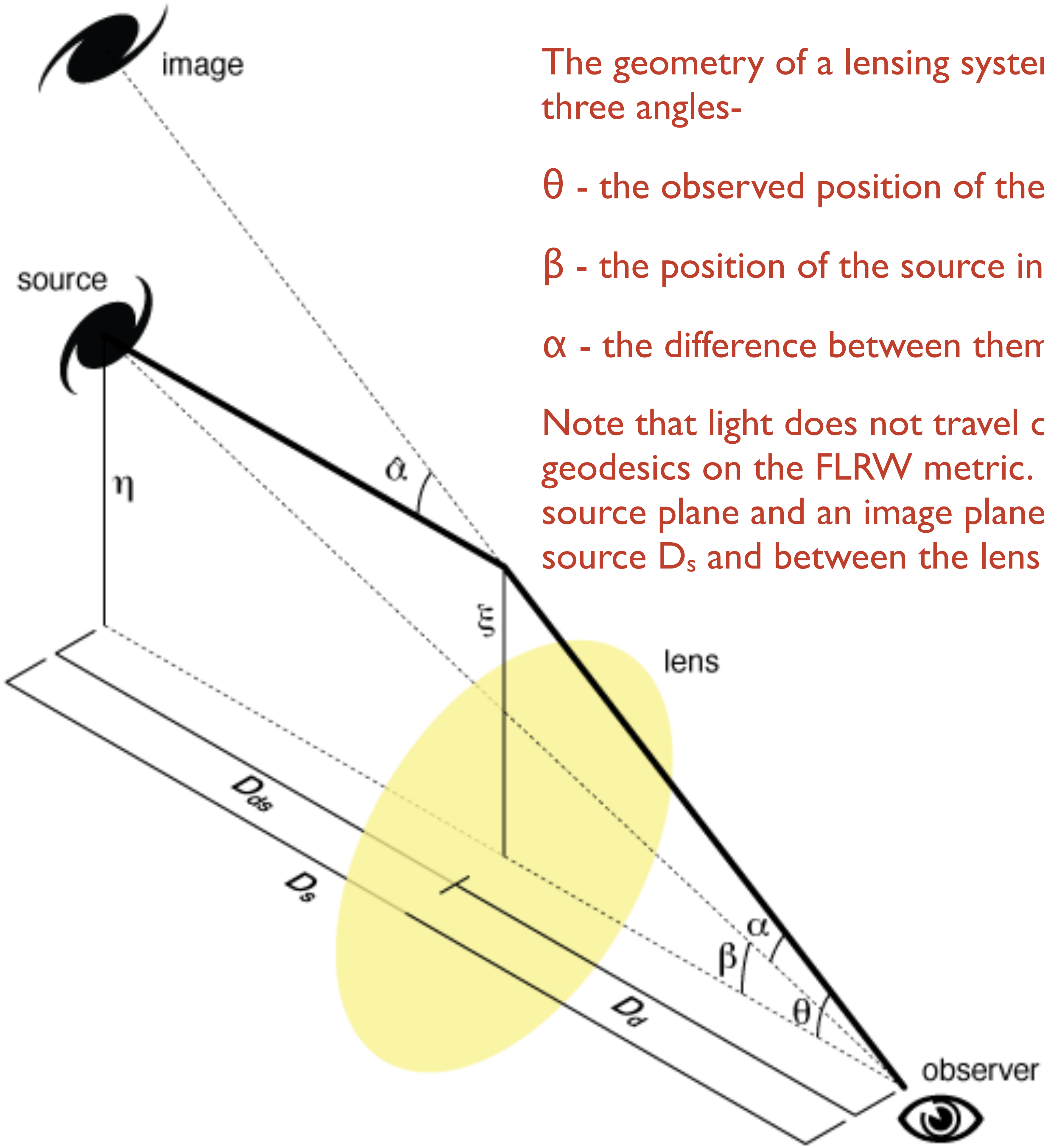
The best intro to gravitational lensing are the lecture notes of Narayan & Bartelmann 1996 available on the archive.

* In general then the deflection angle at a point **ξ** is the sum of contributions from all masses,

$$\vec{\hat{\alpha}}(\vec{\xi}) = \frac{4G}{c^2} \int \frac{(\vec{\xi} - \vec{\xi'})\Sigma(\vec{\xi'})}{|\vec{\xi} - \vec{\xi'}|} d^2\xi'$$

* In the case of circular symmetry we have a form of Gauss's Law and the deflection angle is

$$\vec{\hat{\alpha}}(\xi) = \frac{4GM(\xi)}{c^2\xi}$$



The geometry of a lensing system can be expressed in terms of three angles-

θ - the observed position of the image

β - the position of the source in the absence of lensing

α - the difference between them

Note that light does not travel on straight lines, but along null geodesics on the FLRW metric. Also there is a lens plane, a source plane and an image plane. A distance to the lens D_l, to the source D_s and between the lens and source D_{ls}. We then have

$$\vec{\alpha} \equiv \frac{D_{ls}}{D_s}\vec{\hat{\alpha}}$$

THE LENS EQUATION

Then the lens equation can be written as

$$\vec{\beta} = \vec{\theta} - \vec{\alpha}(\vec{\theta})$$

given the image position, the mass distribution and the relative distances one can solve for **α** and find **β**.

We can define a critical surface density Σ_{cr} by

$$\Sigma_{cr} = \frac{c^2}{4\pi G} \frac{D_s}{D_l D_{ls}}$$

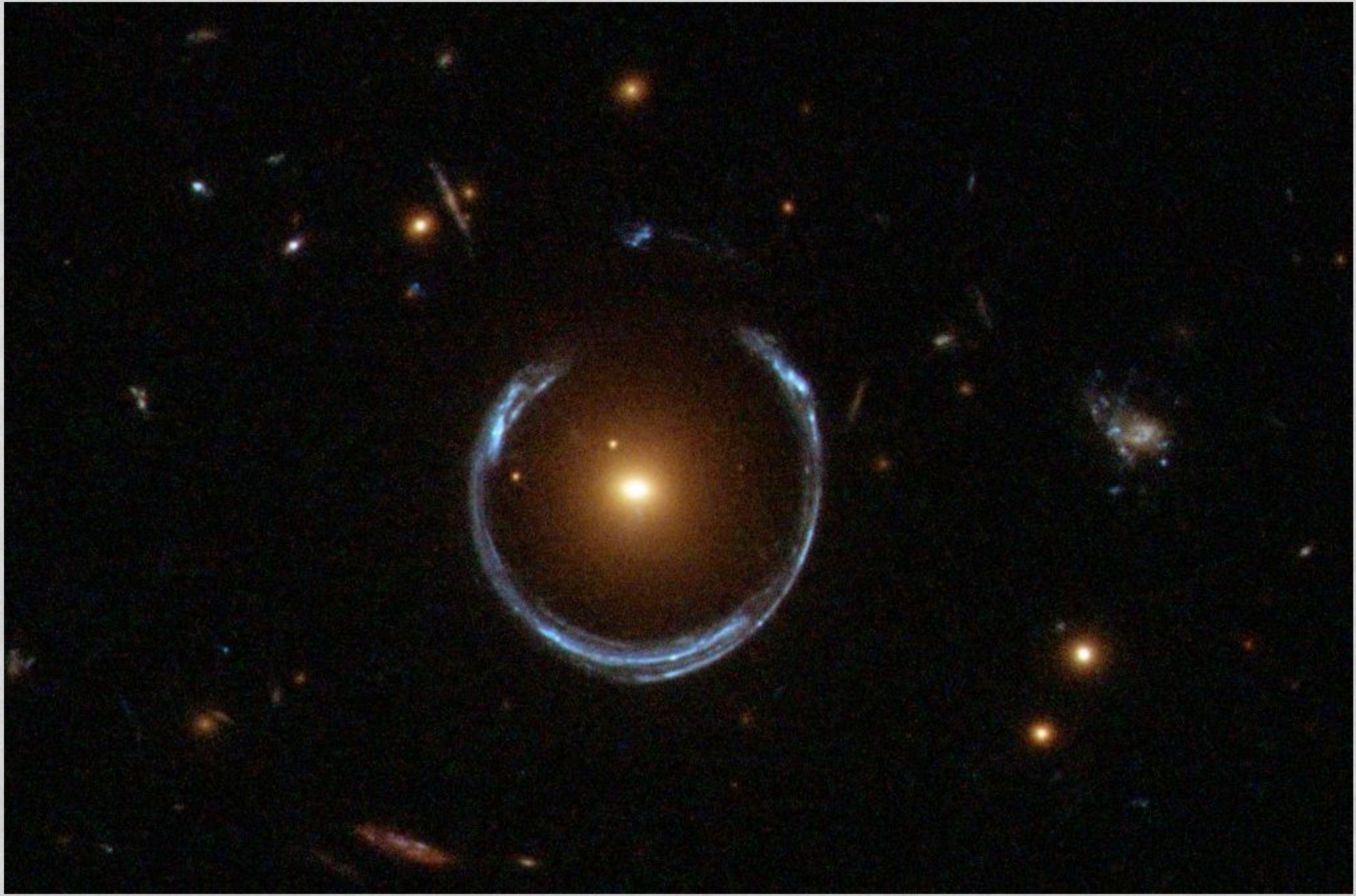
EINSTIEN RADIUS

For a mass distribution with circular symmetry the lens equation gives

$$\beta(\theta) = \theta - \frac{D_{ls}}{D_l D_s} \frac{4GM(\theta)}{c^2 \theta}$$

For β=0 we get a solution for θ which will form a ring around zero. This is called an Einstein ring and its radius is given by

$$\theta_E = \sqrt{\frac{4GM(\theta_E)}{c^2} \frac{D_{ls}}{D_l D_s}}$$



We never get perfect circular symmetry, but sometimes we get close.

THE LENSING POTENTIAL

We can define an appropriately scaled lensing potential from the Newtonian potential.

$$\psi(\vec{\theta}) = \frac{D_{ls}}{D_l D_s} \frac{2}{c^2} \int \Phi(D_l \vec{\theta}, z) dz$$

This potential has the useful properties of

$$\nabla \psi = \vec{\alpha}$$

$$\nabla^2 \psi = \frac{2}{c^2} \frac{D_l D_{ls}}{D_s} 4\pi G \Sigma(\vec{\theta}) = 2 \frac{\Sigma(\vec{\theta})}{\Sigma_{cr}} \equiv 2\kappa(\vec{\theta})$$

Where we have introduced a dimensionless surface density called the *convergence*, κ. Note that this is simply the 2D version of Poisson's equation.

CONVERGENCE

Therefore we can write the potential and deflection angle as integrals of the convergence.

$$\psi(\vec{\theta}) = \frac{1}{\pi} \int \kappa(\vec{\theta}) \ln |\vec{\theta} - \vec{\theta}'| d^2\theta'$$

$$\vec{\alpha}(\vec{\theta}) = \frac{1}{\pi} \int \kappa(\vec{\theta}) \frac{\vec{\theta} - \vec{\theta}'}{|\vec{\theta} - \vec{\theta}'|^2} d^2\theta'$$

Lensing is a form of mapping, knowing κ, every point θ in the lens plane can be mapped to a β in the source plane.

THE JACOBIAN

The local properties of the mapping are described the Jacobian.

$$\frac{\partial \vec{\beta}}{\partial \vec{\theta}} = \delta_{ij} - \frac{\partial \alpha_i}{\partial \theta_j} = \delta_{ij} - \frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j}$$

defining $\frac{\partial^2 \psi}{\partial \theta_i \partial \theta_j} \equiv \psi_{ij}$

we can define $\kappa = \frac{1}{2}(\psi_{11} + \psi_{22})$

$$\gamma_1 = \frac{1}{2}(\psi_{11} - \psi_{22}) \equiv \gamma \cos(2\phi) \qquad \gamma_2 = \frac{1}{2}(\psi_{12}) \equiv \gamma \sin(2\phi)$$

THE JACOBIAN

Then the Jacobian matrix A can be written as

$$A = (1-\kappa)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \gamma \begin{bmatrix} \cos(2\phi) & \sin(2\phi) \\ \sin(2\phi) & -\cos(2\phi) \end{bmatrix}$$

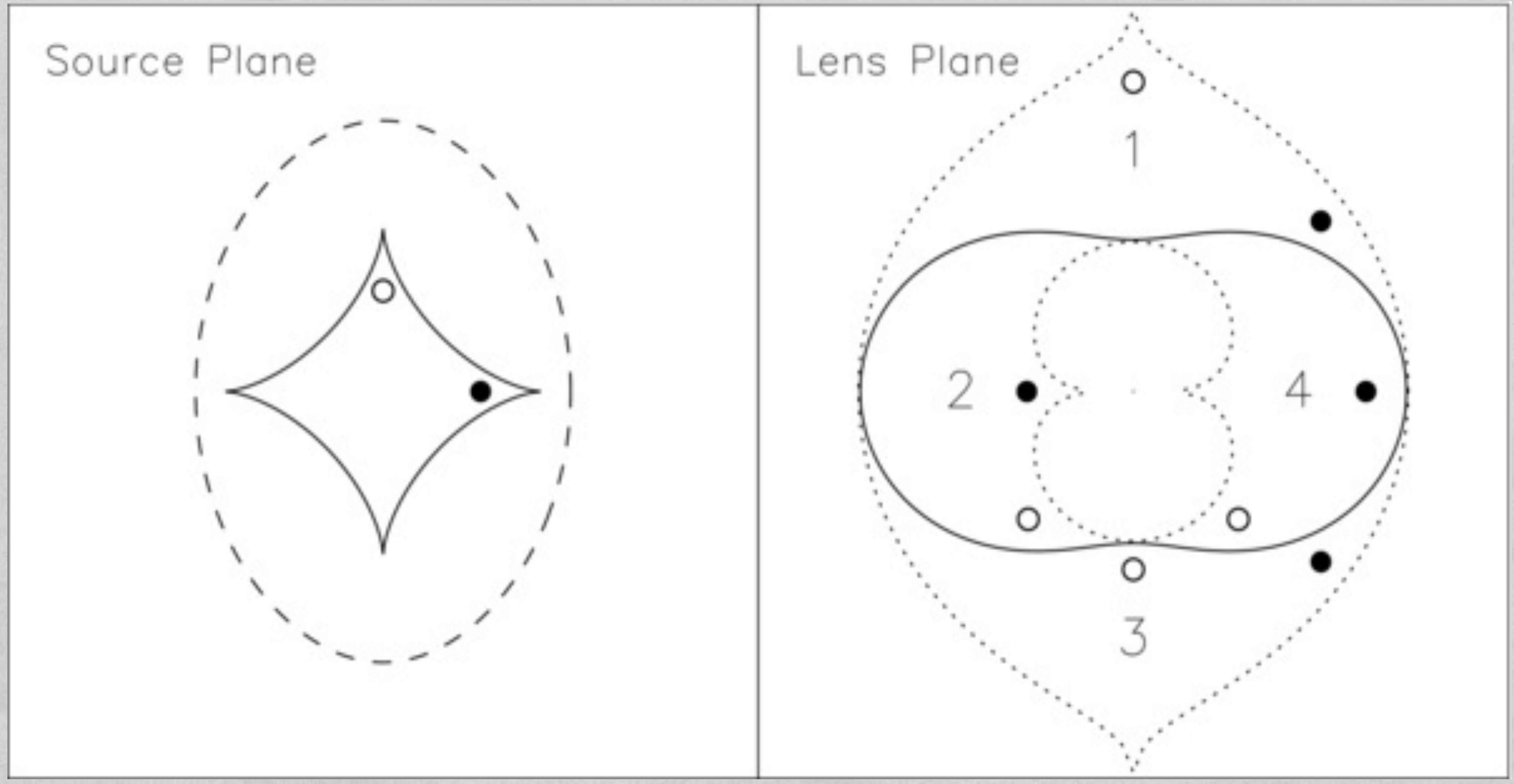
and we can see what the *convergence*, κ, and the *shear* γ do to our source. Convergence by itself magnifies the image in an isotropic fashion, shear creates a distortion along an angle φ. The total magnification is given by

$$\mu = \frac{1}{\det A} = \frac{1}{(1-\kappa)^2 - \gamma^2}$$

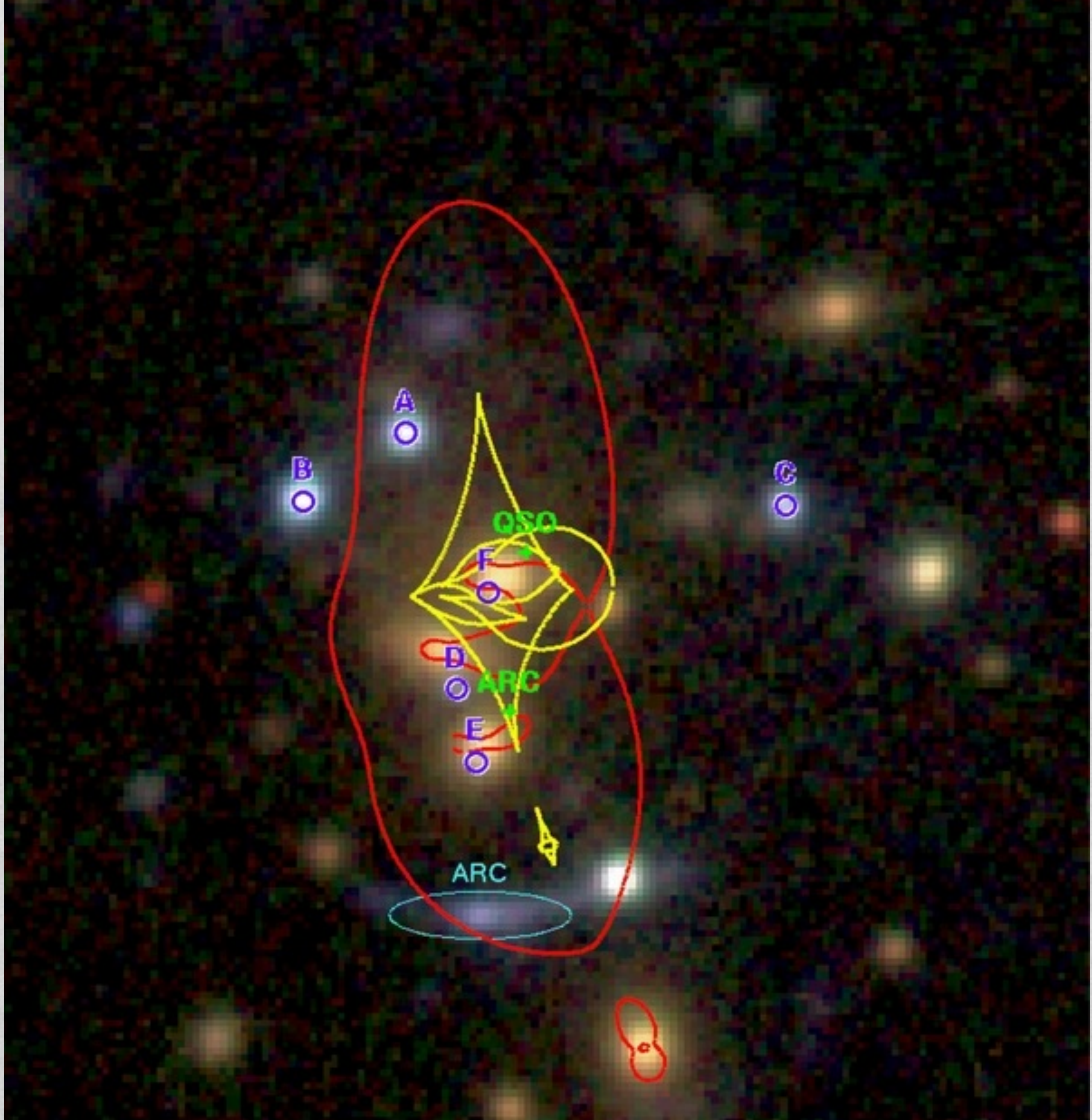
When det A=0 the magnification is infinite, in the case of no shear this happens for κ=1. This means a discontinuity in the mapping.

CAUSTICS AND CRITICAL CURVES

The global structure of the lensing can be drawn out with critical curves and caustics. These are the lines of det A=0. In the source plane they are called caustics, in the image plane critical curves. They delineate the discontinuous part of the mapping.

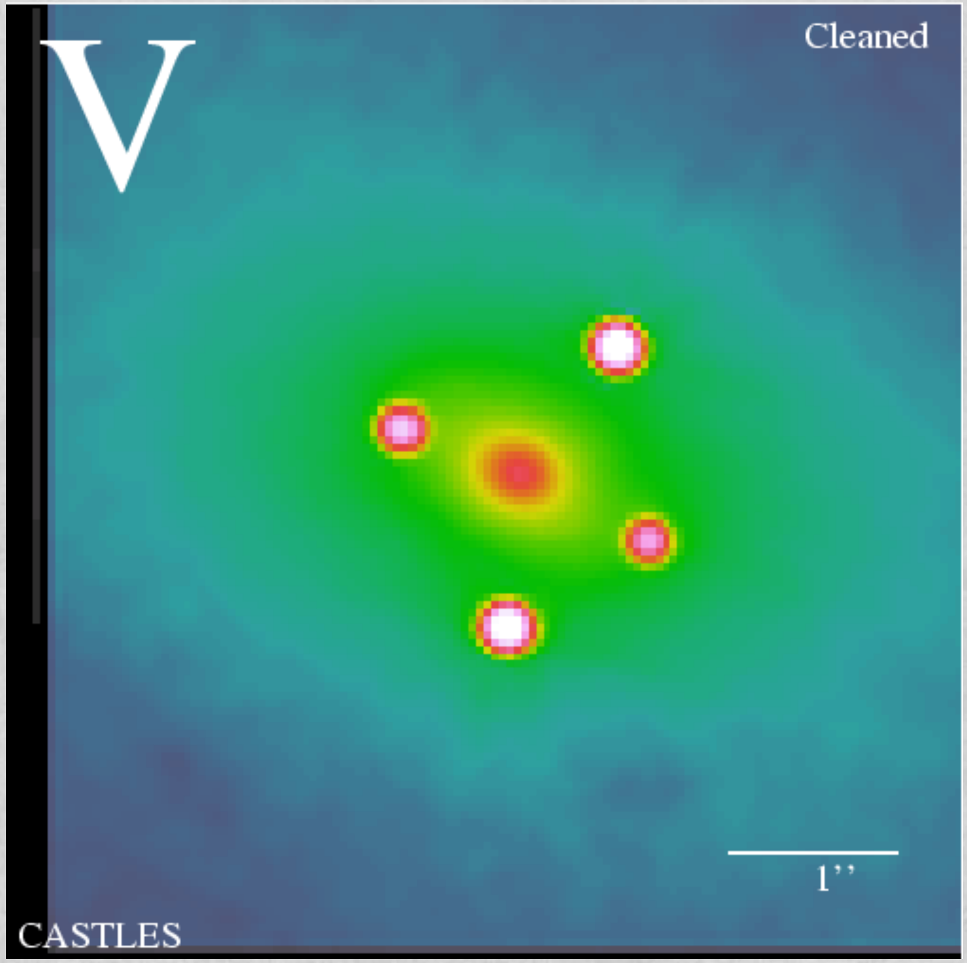


In real mass distributions they can be very messy. When a source crosses a caustic the number of images increases by two. Many lenses however have an even number of images because one image is highly demagnified.

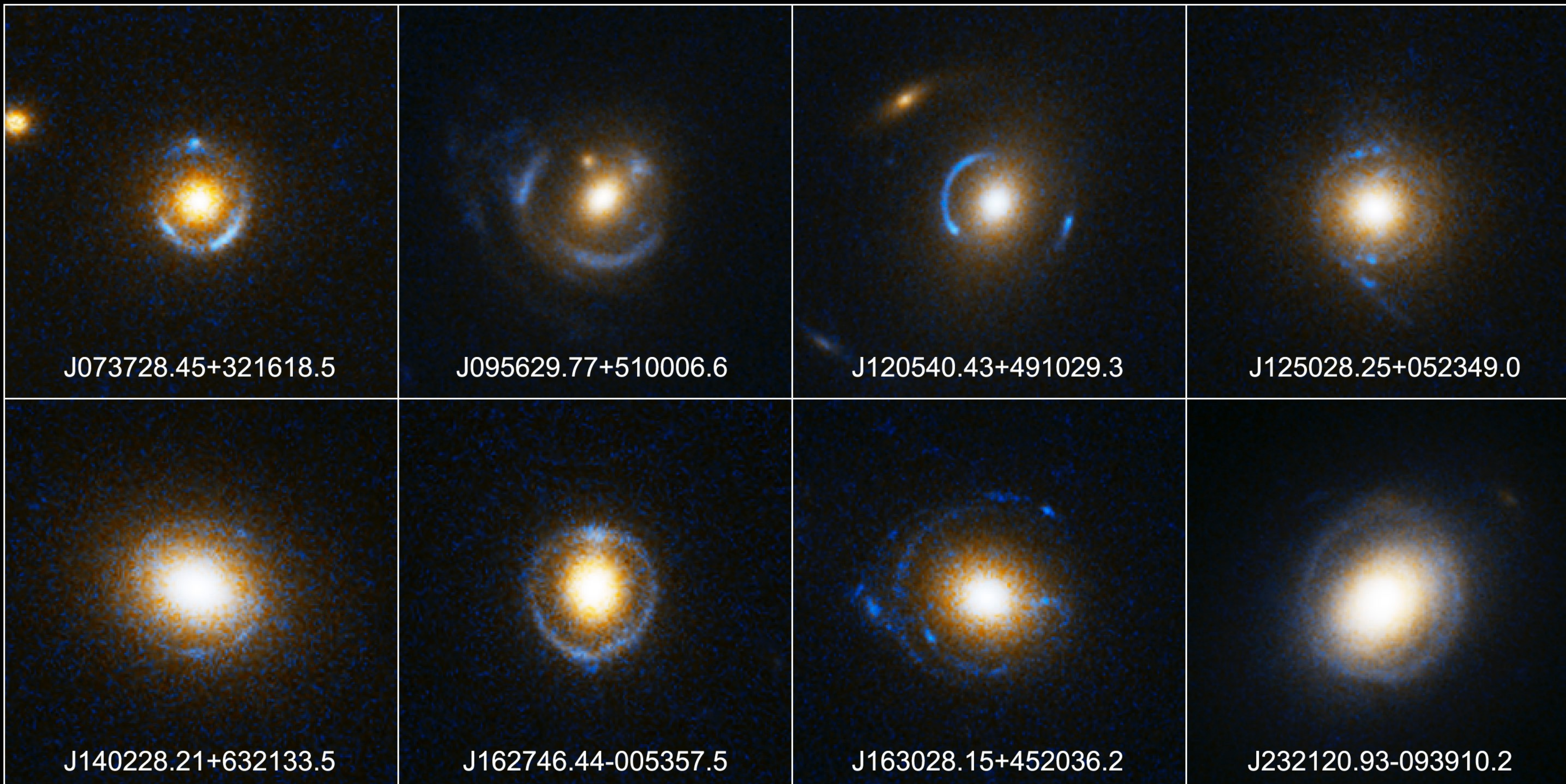


STRONG LENSING

Cases where the distortions do to lensing are evident are called strong lenses. In these cases the source is near a caustic, thus highly distorted or multiple images form. This happens often in clusters and rarely but sometimes (if the alignment is correct) by massive galaxies.



Gravitational lensing in galaxy cluster Abell 2218. NASA, A Fruchter and the ERO Team (STScI).



Einstein Ring Gravitational Lenses

Hubble Space Telescope • Advanced Camera for Surveys

NASA, ESA, A. Bolton (Harvard-Smithsonian CfA), and the SLACS Team

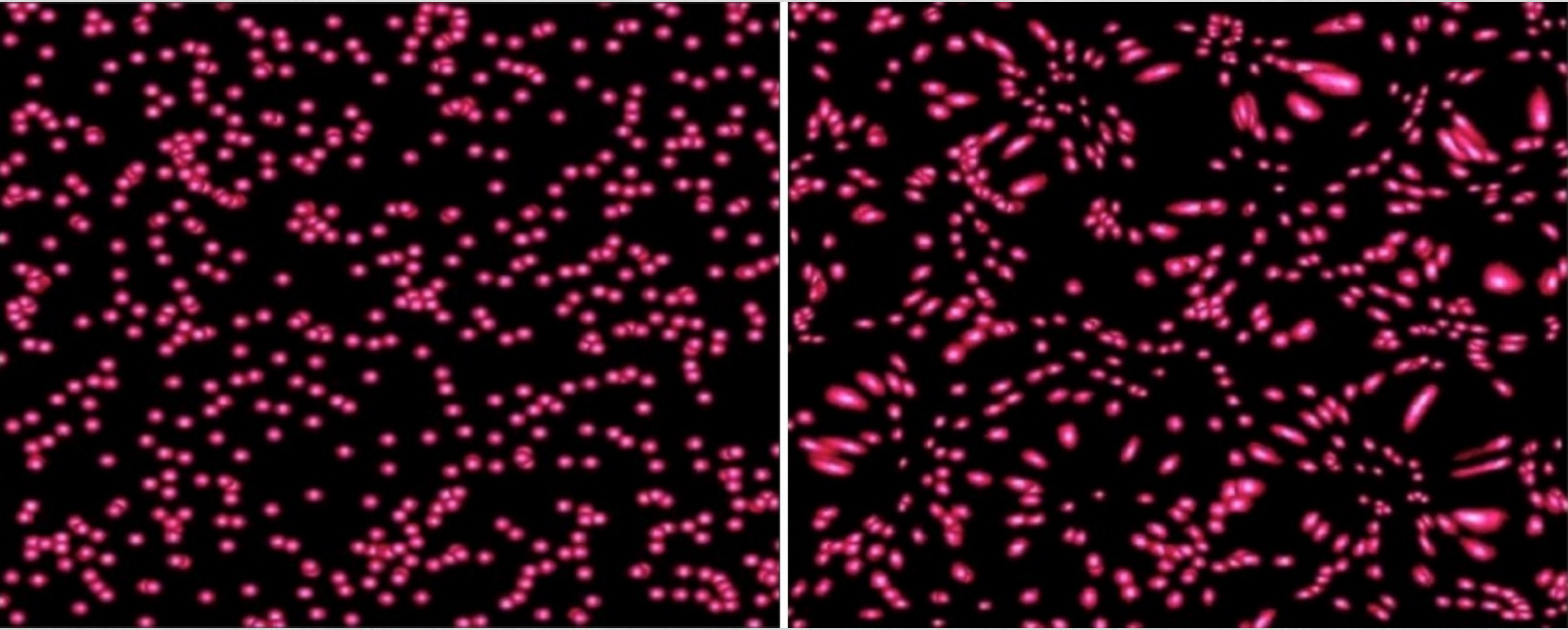
STScI-PRC05-32

MICRO-LENSING

Micro-lensing refers to situations where the deflection angle is far to small to observe. However, the magnification is large and observable. Micro-lensing can therefore only be observed in situations where the source moves fast enough that the change in magnification can be observed. Besides objects in our galaxy micro-lensing can also be observed in strong lenses which are then micro-lensed by stars or other structures moving in the lens potential.

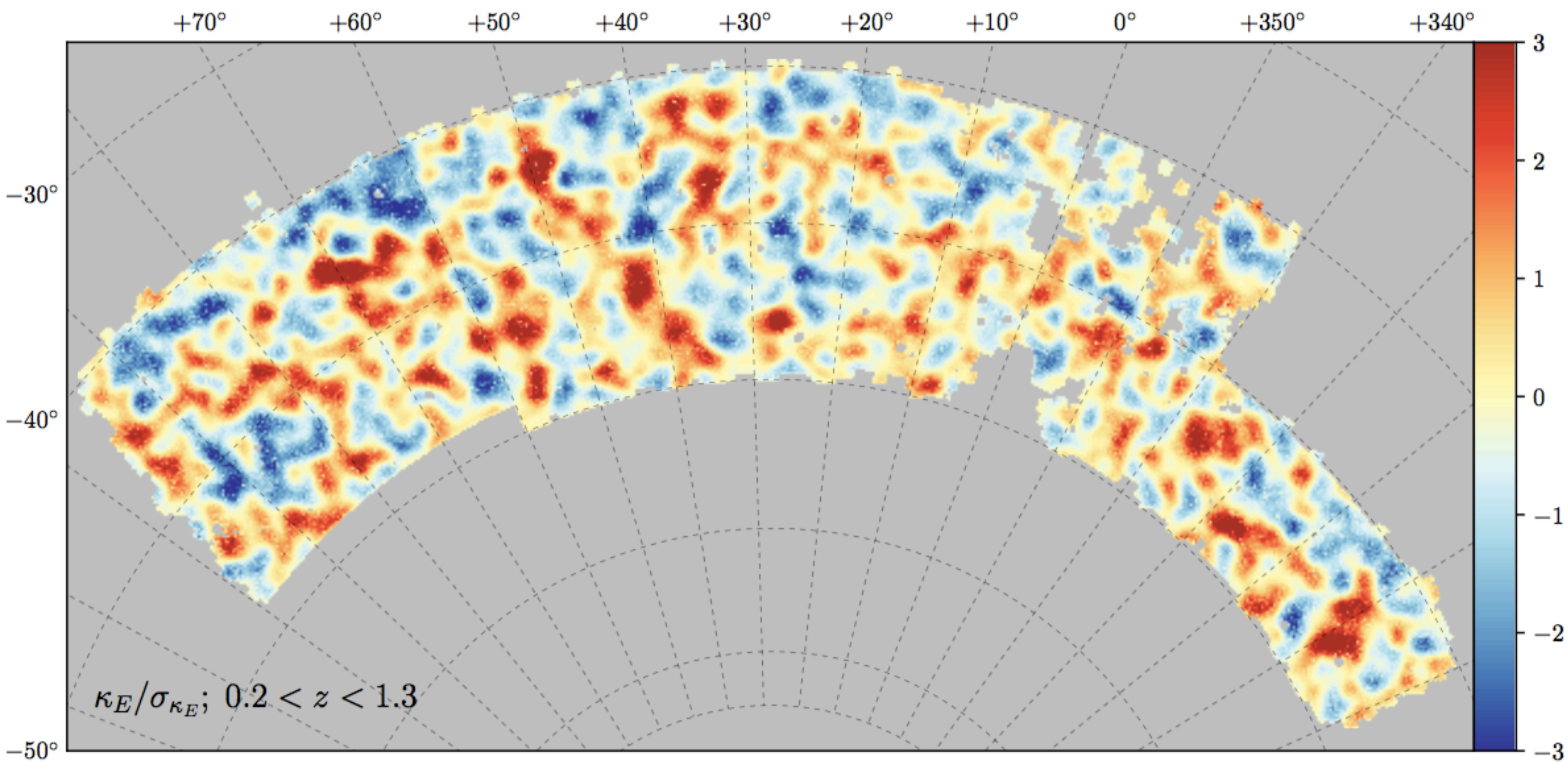
WEAK LENSING

Weak lensing refers to the situation where the deflection angles are small and weak ($\kappa << 1$) so distortions are small. In this case lensing can only be identified statistically. What one does is measures the shapes of all galaxies and determines a shear field from the correlation of the shapes. The mass distribution can be recovered from just the shear field. Weak lensing can measure the masses of clusters, the average halo masses of stacked galaxies and the large scale mass fluctuations in the Universe.



A simulated example. The image on the left is unlensed, on the right distortions are introduced (much larger than in reality). However your eye can pick out the correlated shear field, made much easier since the objects all start as circles. This gives you the idea of how weak lensing works.

DENSITY FLUCTUATIONS FROM GRAVITATIONAL LENSING



This map shows not fluctuations in number of galaxies, but in mass as measured from weak gravitational lensing (DEES)

FUTURE PROSPECTS

Current measurements constrain the ΛCDM cosmological model to a few percent accuracy. The goal of continuing measurements are mostly to measure other parameters (the neutrino mass, gravity waves from inflation) and to break the model. Breaking the model means showing that some assumption in the model is wrong. This could be non Gaussian fluctuations, varying w, or nonstandard gravity.

CURRENT STATUS

Right now the best cosmological constraints come from combining supernova Type 1a, the CMB power spectrum and correlation functions (often just called large scale structure and the BAO part is the most important).

This gives the high precision values that are often quoted. However, the real goal now is to find tension between these measurements or other measurements that create problems and can lead to new physics.

COMPOSITION OF THE COSMOS