COSMIC INFLATION

Week 6

OUTLINE

- Problems with the standard model
- An inflationary phase
- Generating fluctuations
- The end of inflation

FLRW-COSMOLOGY

- The Universe's expansion is governed by the Friedmann equations that give a and z as a function of time.
- This model has three parameters, the matter density, Ω_m, the dark energy density, Ω_Λ, and the Hubble constant H_0, or current time t_0. The amount of curvature, Ω_K, is then given by $1-\Omega_m - \Omega_\Lambda$.
- The model predicts Big Bang Nucleosynthesis which depends on another parameter the baryon-to-photon ratio, η_γ.
- The model also predicts that there was a time when photons were coupled to baryons that ended leaving a Cosmic Microwave Background. From the CMB we can see that deviations from homogeneity are small $\sim 10^{-5}$.

PROBLEMS

1. The Magnetic Monopole Problem
1. The Flatness Problem
2. The Horizon Problem
3. The problems with fluctuations

MAGNETIC MONOPOLES

- For particle physicists working on grand unification theories (GUT) the fact that we don't observe any magnetic monopoles seems like a problem.
- The point is that magnetic monopoles or other exotic particles are often produced in these models, and they would become non relativistic early and should dominate the matter density today, but we don't see any.
- For most people this hardly seems like much of a problem, especially compared to the other ones, but it was actually the original motivation for inflation.

GRAND UNIFIED THEORY

- The success of electroweak unification by Weinberg, Salam and Glashow was one of the crowning achievements of particle physics in the 80s.

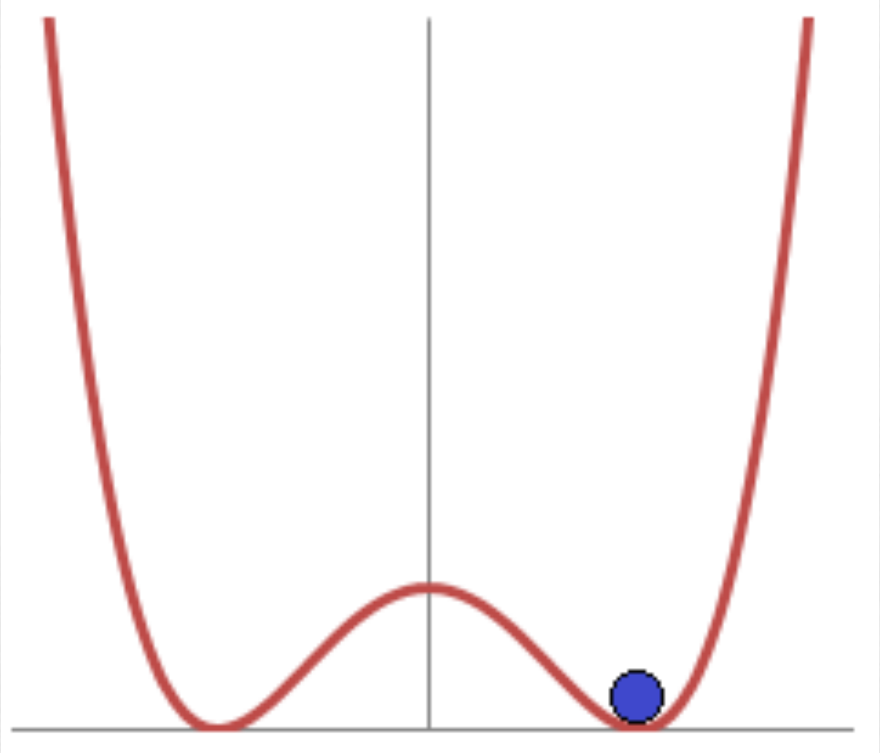
- This theory stated that the electromagnetic and weak force were different because the bosons in the weak force were massive (80-90 Gev) while the photon is massless.

- At energies greater than ~100Gev this difference would go away and you would have one unified electroweak force.

- This unification is often written in terms of the symmetries of the theory, $H \rightarrow SU(2) \times U(1)$

SPONTANEOUS SYMETTRY BREAKING

- The electroweak theory is said have spontaneous symmetry breaking which means that the overall theory has the symmetry, but that it is broken by the ground state not being symmetric.



- An example is the potential shown to the right. Clearly this potential is symmetric around x=0, but the object has happened to fall on the right side, breaking the symmetry.

GUT AND TOE

- Based on the success of the electroweak model people turned to combining electroweak with the strong force (GUT) and even gravity (TOE).

- GUT proposed a unified symmetry, $G \rightarrow H \rightarrow SU(3) \times SU(2) \times U(1)$, but this has turned out to be rather hard to make work. Early models had some predictions, like the decay of the proton, that have been ruled out by experiment. Now days people mostly focus on adding other things, super symmetry, strings, etc. before the GUT scale.

- Nevertheless, the general idea that the Universe would go through many symmetry breakings as it cools remains.

PHASE TRANSITION

- A phase transition can be of two types:
  - First-order transitions occur through the formation of bubbles. The bubbles expand and collide until only the new phase is left.
  - Second-order transitions occur smoothly with the old phase transforming into the new phase in a continuous manner.
- If any of the symmetry breaking we have discussed is first-order than we would expect topological defects occur.

TOPOLOGICAL DEFECTS

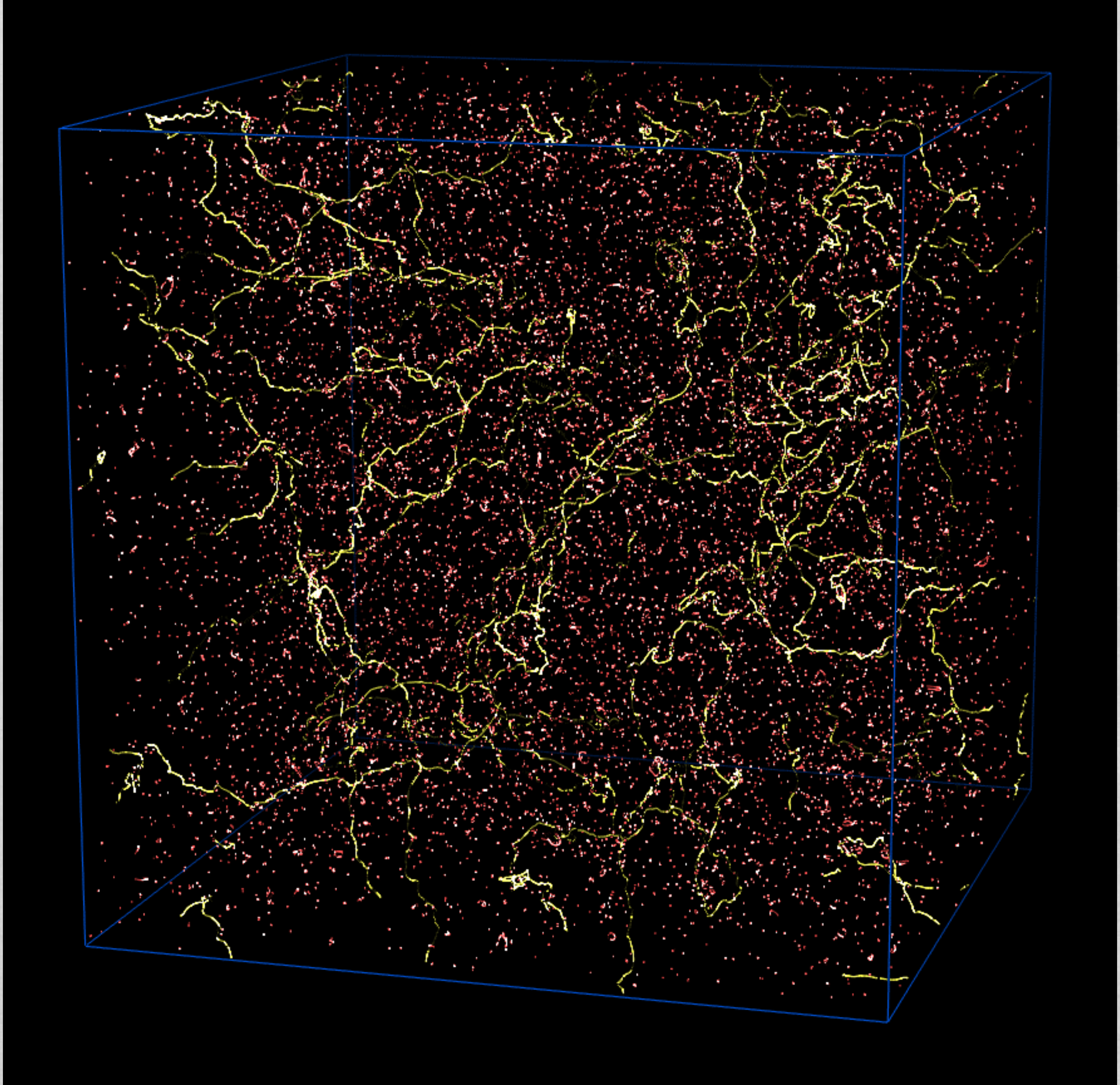
- Topological defects are matter that is trapped in the old phase, prevented from making the phase transition. Depending on the type of symmetry you get a different type of defect.
    - Domain Walls are the result of discrete symmetry breaking. Domain walls effectively partition the universe into cells. The gravitational field of a domain wall is repulsive.

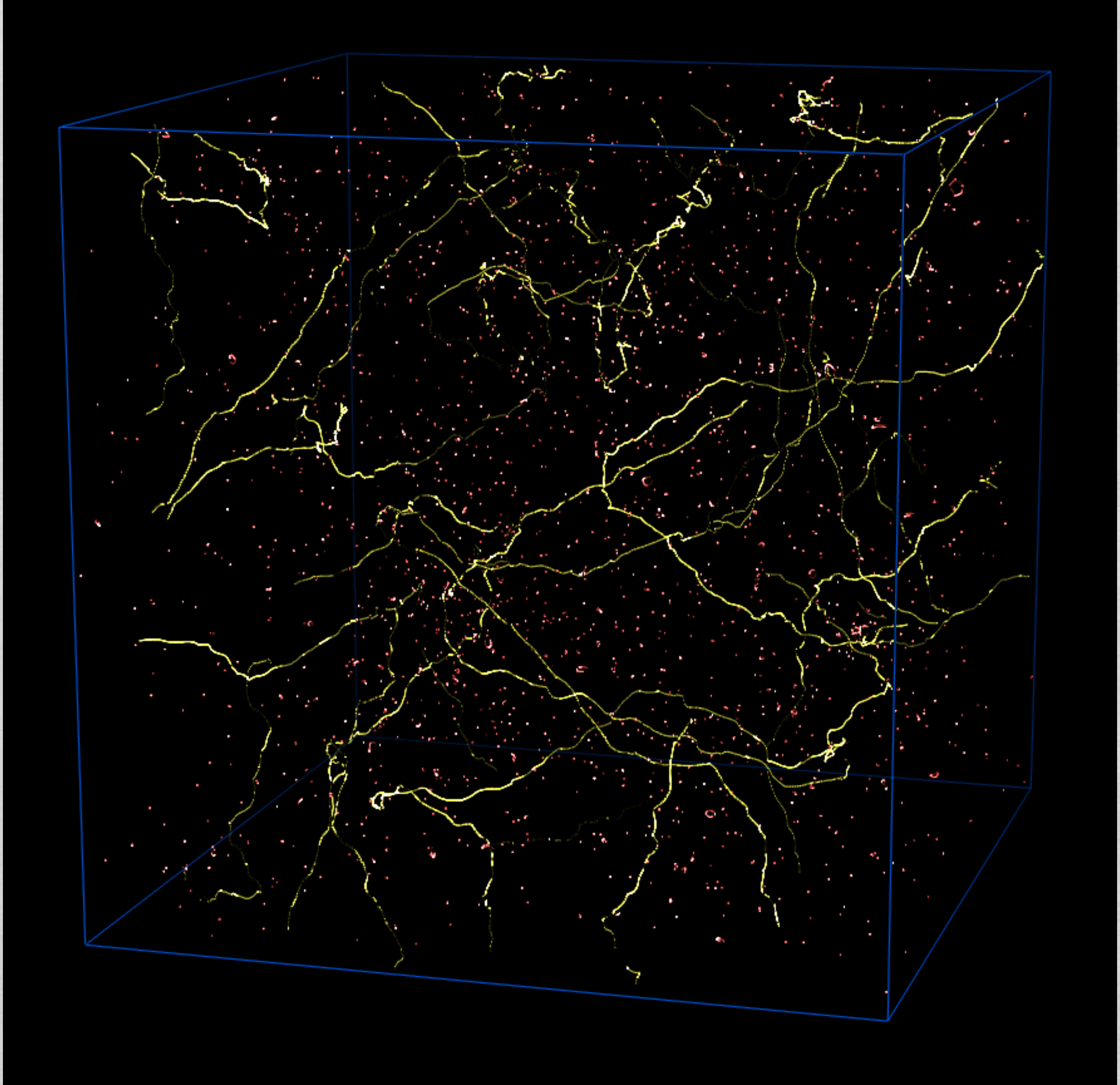
TOPOLOGICAL DEFECTS

- Cosmic strings are one-dimensional objects that form with cylindrical symmetry is broken. They can form from GUT or electroweak symmetry breaking. They are very small, thickness less than 10^9 of a hydrogen atom, but can be massive and cause structure formation.

- Monopoles form if spherical symmetry is broken. They are very massive and carry magnetic charge. The existence of monopoles is an inevitable prediction of GUT theory so their nondetection is a problem.

- Textures are more complicated symmetries, they tend to be unstable.





MAGNETIC MONOPOLES

- So if their is a broken GUT symmetry their should be magnetic monopoles which can be detected.
- The magnetic monopole problem is why don’t we detect any magnetic monopoles.
- Note that why cosmological defects have never been seen, this same physics is well studied in condensed matter systems. If symmetry breaking occurs then we expect cosmological defects to exist.

THE FLATNESS PROBLEM

- We currently find that $|\Omega_K| < 0.01$, but even when we only knew $0.1 < \Omega_m < 1.0$, this raised a fine tuning problem.

- If $\Omega_m \neq 1$ it evolves farther and farther away from one as the Universe ages. Thus if Ω_m is anything close to one today it must have been ridiculously close to one at early times.

- For example at the Planck scale ($t_P \sim 5\times10^{-44}$s) the curvature $\Omega_K < 10^{-60}$ for $\Omega_m > 0.1$ today.

THE FLATNESS PROBLEM

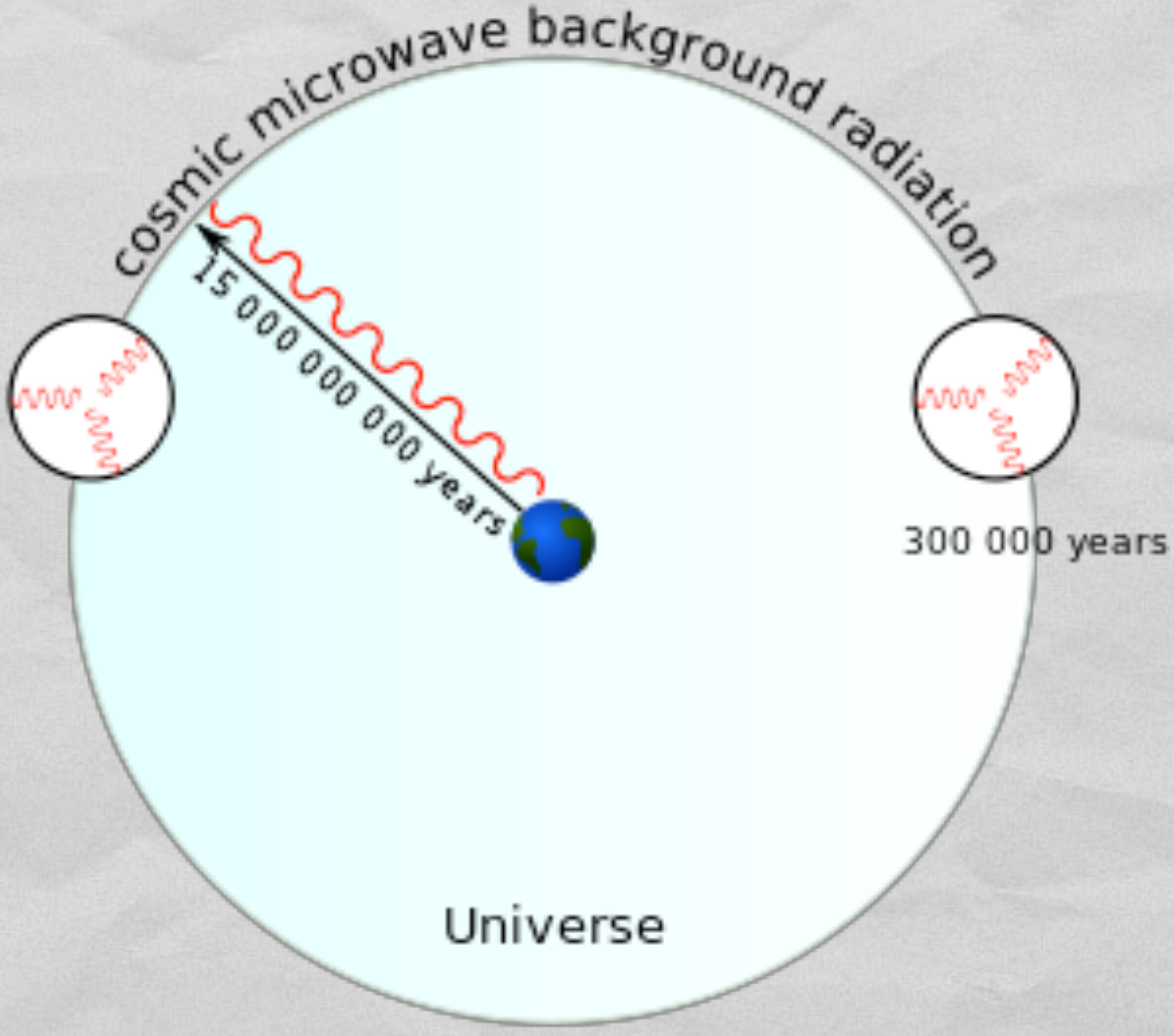
- Ryden gives a nice example of how small this is. The Sun’s mass would change by 10^{-60} by removing 2 electrons.
- Such fine tuning of the matter density requires an explanation. Nothing in our model so far can account for the apparent flatness of the Universe, which is dubbed the ‘flatness’ problem.

THE HORIZON PROBLEM

- The horizon problem has to do with the extraordinary uniformity of the CMB, that it is a perfect blackbody at 2.728K with rms fluctuations of 0.000018K.
- Recombination takes place 300,000 years after the Big Bang so only particles within 600,000 lyr of each other could possible have shared any information.
- How do particles which are as much as 26 Glyr apart know to have the same temperature.

THE HORIZON PROBEM

The CMB's uniformity is a problem because the particle horizon at the time was much much smaller than the scale over which we see uniformity.



DENSITY FLUCTUATIONS

- Finally, we observe density fluctuation today and in the CMB, but we have not addressed them in our theory. Neither their origin or their spectrum. Topological defects is one possible origin of fluctuations, we will see inflation is another.
- In addition we see structures today on the scale of 100 Mpc and at the same co-moving size in the CMB. But gravity is a very weak force. It seems that there are correlations in the density field that are larger than the horizon scale.

COSMOLOGICAL PRINCIPLE

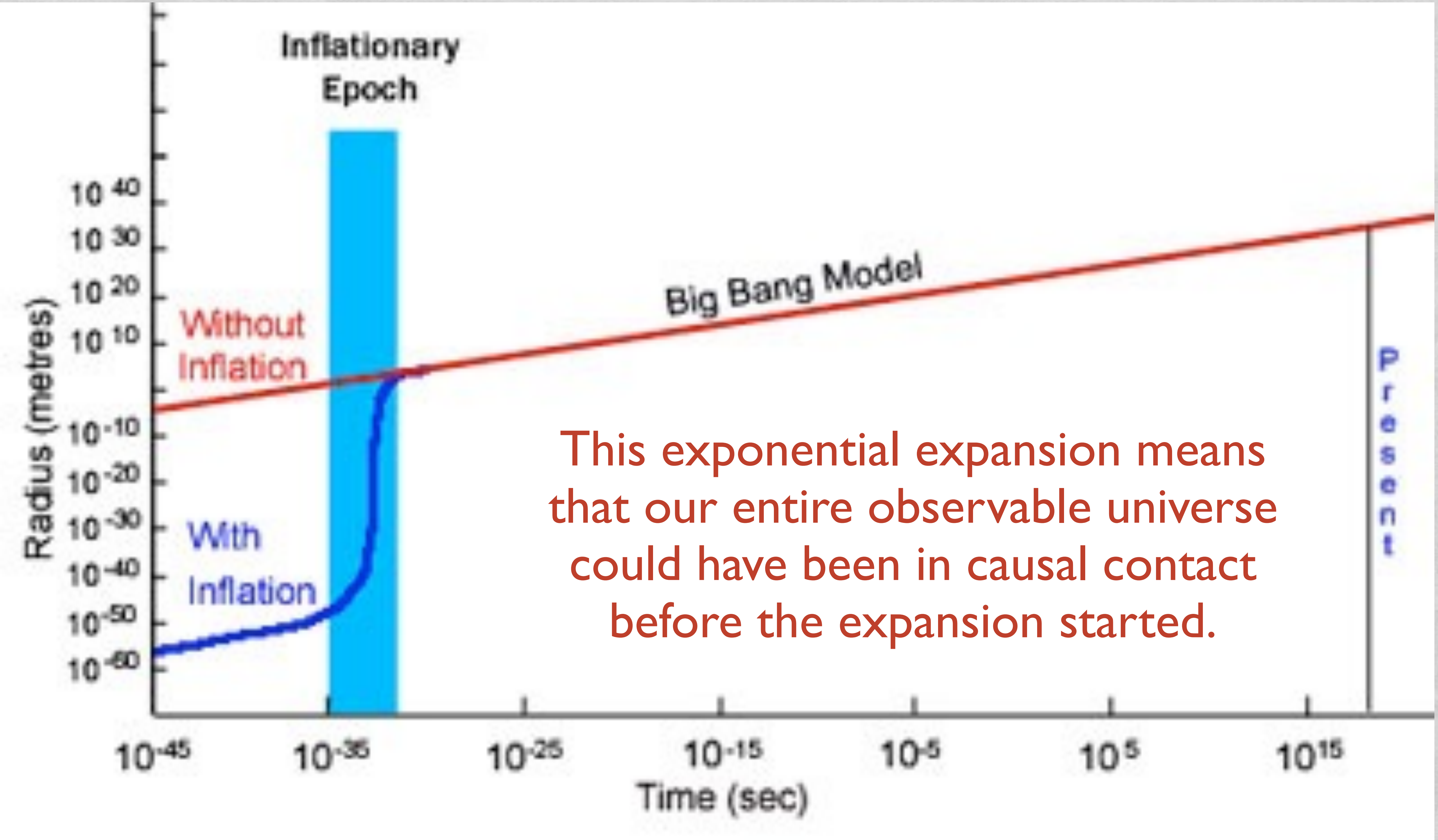
- Another way to look at this is that the cosmological principle is too good.
- We started our theory of cosmology assuming homogeneity and isotropy, thinking it was mostly true.
- But it turns out at early times those assumptions have to be true with amazing accuracy.
- There is a naturalness problem to our cosmological model, like finding a pencil standing on the point. A perfectly valid solution to Newton's equations, but exceedingly unlikely.

INFLATION AS A SOLUTION

INFATIONALRY SCENARIO

- In 1980, Alan Guth proposed that the Universe might have undergone a period of rapid expansion early on while undergoing phase transitions during GUT symmetry breaking.
- These phase transitions cause a false vacuum which acts like a cosmological constant until the true vacuum is reached.
- While dominated by Λ_i, the Friedmann equations give $a(t) = e^{H_i t}$ as we have seen. So during this phase the Universe undergoes a rapid period of expansion.

EXPONENTIAL EXPANSION



This exponential expansion means that our entire observable universe could have been in causal contact before the expansion started.

E-FOLDING

- During the period of inflation the growth rate of the Universe is exponential. So the total increase is

$$\frac{a(t_f)}{a(t_i)} = e^N \qquad \text{where} \qquad N \equiv H_i(t_f - t_i)$$

- As an example consider inflation that starts at the GUT scale $t_{GUT} \sim 10^{-36}$ s. Then $H_i \sim 1/t_{GUT} = 10^{36}$. If $N \sim 100$ then $t_f = 10^{-34}$ s and

$$\frac{a(t_f)}{a(t_i)} \sim e^{100} \sim 10^{43}$$

- So in 10^{-34} s the Universe expands by 10^{43}.

PROBLEMS SOLVED?

- We can see how this period of rapid expansion would solve some of our stated problems.
- Any monopoles, topological defects, domain walls or other exotic objects would be so diluted by the expansion we would never expect to find them.
- Our whole Universe could have been in causal contact before the expansion, then the Universe expanded so fast those regions were moved out of our horizon and some of them are only coming back in today.
- And a Universe dominated by dark energy is driven towards $\Omega = 1$ the longer it is dark energy dominated.

THE FATE OF DARK ENERGY UNIVERSES

The Friedmann equation

$$\Omega_K = \frac{kc^2}{a(t)^2 H(t)^2}$$

During exponential expansion

$$\Omega_K \propto e^{-2H_i t}$$

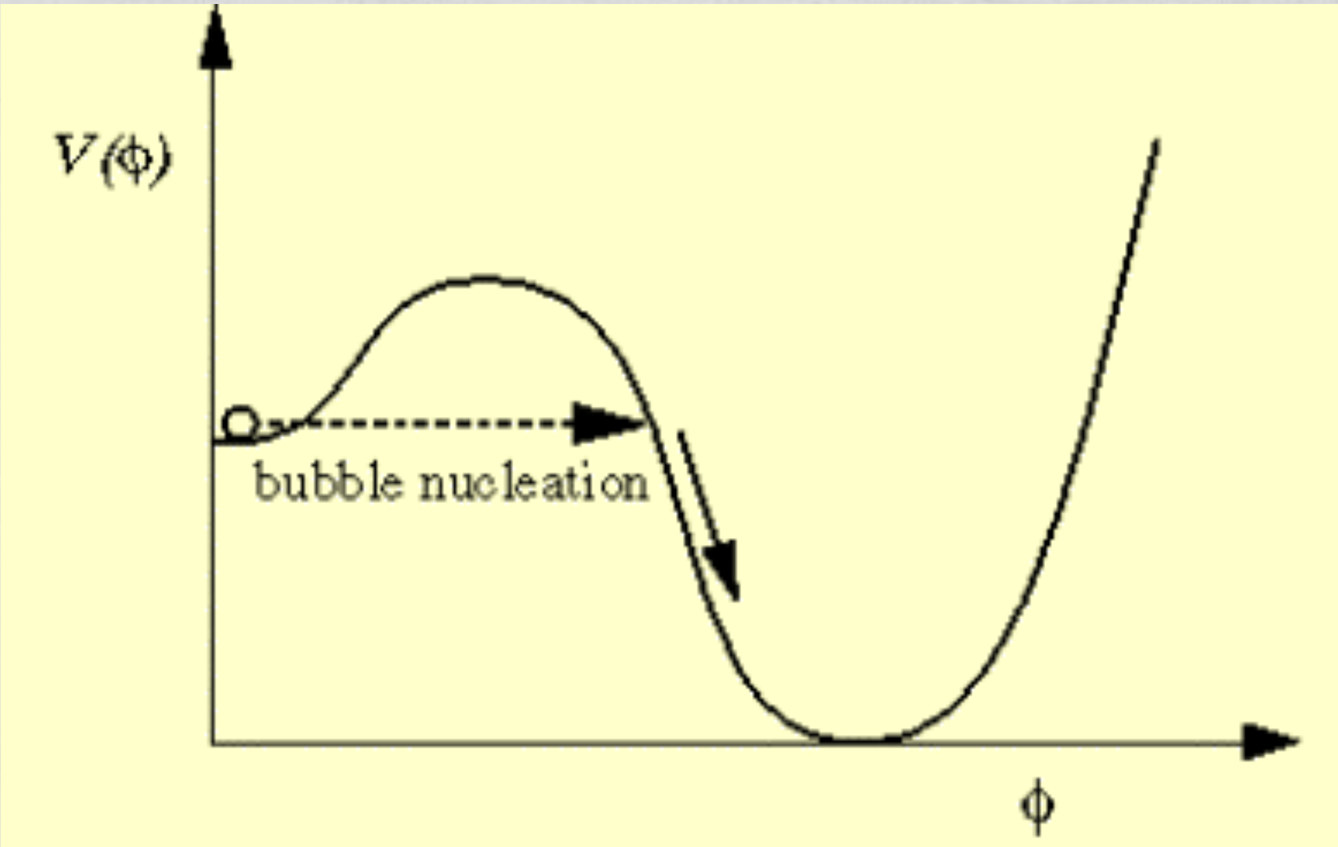
So if we consider inflation that goes lasts N/H_i, then

$$\Omega_K(t_f) = e^{-2N}\Omega_K(t_i)$$

Suppose before inflation the Universe was strongly curved Ω_K~ 1, after inflation the curvature would be e^{-200} ~ 10^{-87}, incredibly flat.

OLD INFLATION

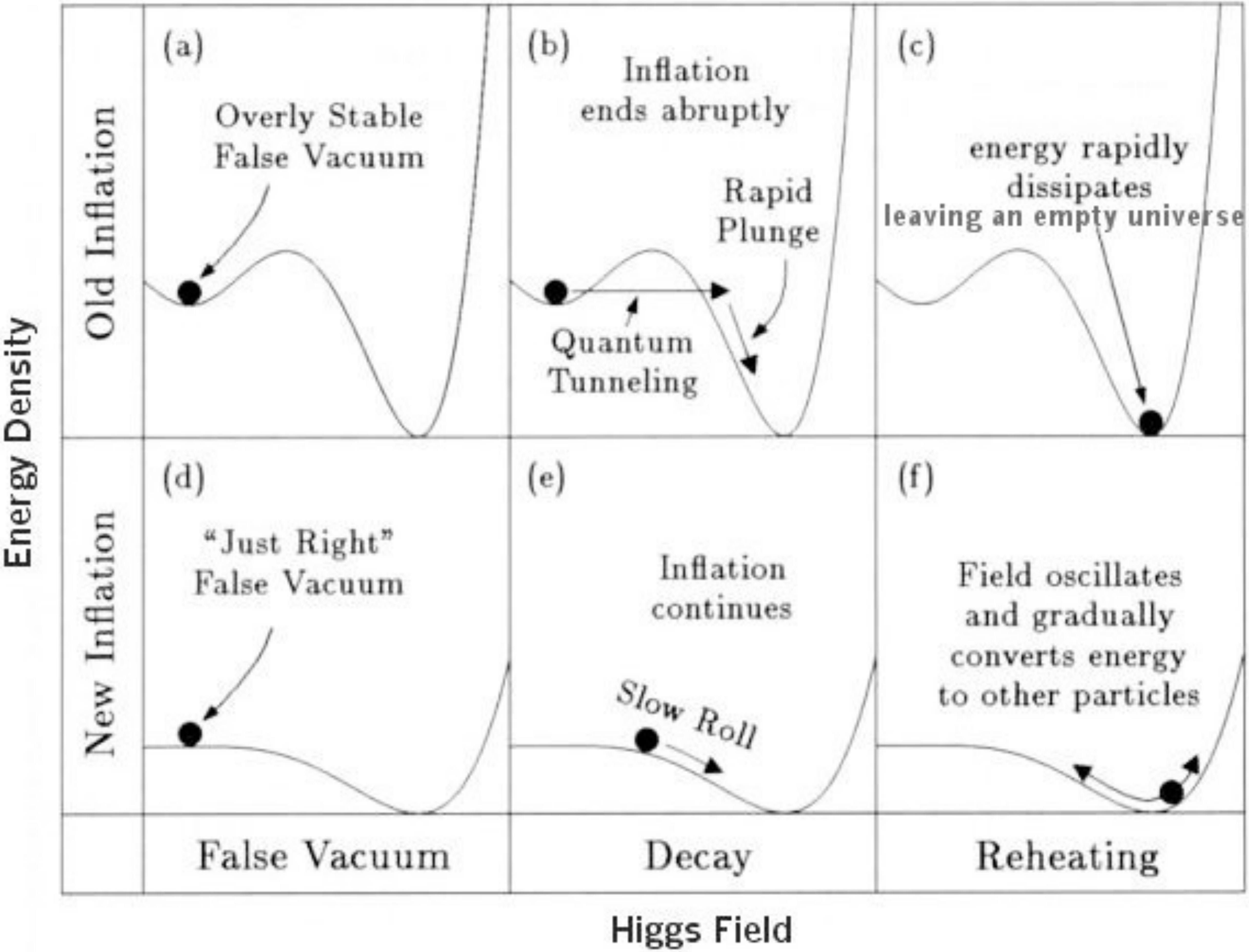
- In the original inflation model of Guth inflation is caused by a scalar field with a local, but not global minimum. Inflation ends when the the field quantum tunnels to the global minimum.
- However, it was soon realized that this model has trouble reheating the Universe. All the energy goes into the bubble walls and can be released when they collide, but this is inhomogenous and expanded away.



This is called the graceful exit problem.

NEW INFATION

- This problem was solved by Linde and by Albrecht and Stienhardt who both proposed slow-roll inflation.
- In this case the inflationary period is not a local minimum but instead a potential with shallow slope such that it takes the field time to reach the minimum.
- When the field falls to the minimum it oscillates, and those oscillations produce particles that reheat the Universe.


Energy Density
Old Inflation
New Inflation
(a)
Overly Stable
False Vacuum
(b)
Inflation
ends abruptly
Rapid
Plunge
Quantum
Tunneling
(c)
energy rapidly
dissipates
leaving an empty universe
(d)
"Just Right"
False Vacuum
(e)
Inflation
continues
Slow Roll
(f)
Field oscillates
and gradually
converts energy
to other particles
False Vacuum
Decay
Reheating
Higgs Field

CHAOTIC INFLATION

- Linde also proposed chaotic inflation. In this model the value of the inflation field undergoes quantum fluctuations.
- In some places it begins the slow roll to reheating the Universe.
- In other places the Universe keeps inflating. Because the rate of inflation is so large even if only a few places remain in the inflationary state, they quickly grow to be as large as all the regions that fell out of inflation.
- Thus inflation goes on forever. There are always places where inflation ends and reheating happens, but there is also always places where inflation is ongoing.
- Thus the Universe is eternal and we can answer the question of what happened before inflation by saying more inflation.

OTHER MODELS

- There are now many inflation models, as long as some vacuum energy is created and reheating occurs there really aren't any other constraints on one's model.
- Eternal inflation suggests that the universe starts in the inflation mode and only some parts ever drop out. Our universe is a very tiny part of the whole similar to chaotic inflation.
- Hybrid inflation suggests their are multiple scalar fields, the one that causes inflation is different than the one that ends it.
- There are also attempts to get inflation from string theory and quantum gravity.

AN EXAMPLE

Let's assume there is a scalar field, which we will call the inflaton which has a associated potential V. The the energy density of the field is:

$$\epsilon_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 + V(\phi)$$

If the field is homogenous the pressure is

$$P_\phi = \frac{1}{2}\frac{1}{\hbar c^3}\dot{\phi}^2 - V(\phi)$$

If the field changes very slowly with time (this is the slow roll), such that

$$\dot{\phi}^2 << \hbar c^3 V(\phi)$$

Then the field acts like dark energy

$$\epsilon_\phi \approx -P_\phi \approx V(\phi)$$

Let's look at this field under the fluid equation.

$$\dot{\epsilon} + 3\frac{\dot{a}}{a}(\epsilon + P) = 0$$

This gives

$$\ddot{\phi} + 3H(t)\dot{\phi} = -\hbar c^3 \frac{dV}{d\phi}$$

Homework
Show this is true

AN EXAMPLE

$$\ddot{\phi} + 3H(t)\dot{\phi} = -\hbar c^3 \frac{dV}{d\phi}$$

looks like the motion of a particle in a potential V with a frictional term 3H(t)

If we imagine the field has reached a 'terminal' velocity then $\ddot{\phi} = 0$

and $\dot{\phi} = -\frac{\hbar c^3}{3H}\frac{dV}{d\phi}$ since $\dot{\phi}^2 << \hbar c^3 V(\phi)$ to have a slow roll, we get

$$\left(\frac{dV}{d\phi}\right)^2 << \frac{9H^2V}{\hbar c^3}$$

if the Universe is dominated by inflation the Hubble value is given by

$$H = \left(\frac{8\pi G V}{3c^2}\right)^{\frac{1}{2}}$$

so

$$\left(\frac{dV}{d\phi}\right)^2 << \frac{24\pi G V^2}{\hbar c^5} = 24\pi \left(\frac{V}{E_p}\right)^2$$

AN EXAMPLE

So a field that satisfies the conditions

$$\left(\frac{dV}{d\phi}\right)^2 << \left(\frac{V}{E_p}\right)^2 \quad \text{and} \quad \dot{\phi}^2 << \hbar c^3 V(\phi)$$

can be the source of inflation. These are quite generic requirements so many possible inflation fields have been explored over the years.

In addition there are strong constraints on the nature of the inflation field based on how it reheats and how it forms density fluctuations. The latter is one of the most important predictions of inflation.

REHEATING

- During inflation the vacuum energy of the field dominates the energy density of the universe. All other matter is redshifted into negligible amounts.
- This creates a very ordered universe, their is no thermal randomness in it. In order to get the hot big bang which we have found produces the CMB and BBN we need the inflation field to decay into thermal particles.
- Exactly how this occurs and what temperature the universe reheats to depends on the inflation model. It could be very inefficient and only a small fraction of the energy goes into thermal radiation, or very efficient with almost all the inflaton energy becoming thermal energy.

FLUCTUATIONS

- As we've seen inflation does a tremendous job at erasing any inhomogeneities in the Universe and flatting it out.

- But we know we need some fluctuation to create the structure seen today and in the CMB.

- It was soon realized that because the expansion in inflation is so large, quantum fluctuations on microscopic scales grow to cosmological scales.

- When a fluctuation grows larger than the horizon it gets 'frozen in' and thus fluctuations on very large scales can exist.

FLUCTATIONS

The fluctuations can be expressed as a power spectrum

$$\delta(\vec{r}) \equiv \frac{\rho(\vec{r})}{\bar{\rho}(\vec{r})} - 1 = \frac{V}{(2\pi)^3} \int \delta_{\vec{k}}\, e^{-i\vec{k}\cdot\vec{r}} d^3k$$

where k is a wavenumber that corresponds to a spatial scale.
The power spectrum P(k) is then given by

$P(k) =< |\delta_k|^2 >$ where δ_k is complex $\delta_{\vec{k}} = |\delta_{\vec{k}}| e^{i\phi_{\vec{k}}}$

most inflation models predict that the phases, φ_k, of the different components are independent giving rise to Gaussian fluctuation and that he power spectrum is a power law $P(k) \propto k^{n_s}$

where $n_s \sim 1$

LEGACY

- While inflation seems very successful, in many ways it simply replaces fine-tuning problems (the flatness and horizon problems) with other fine-tuning problems (the values of the inflaton field).
- Inflations greatest legacy is a prediction of the original density fluctuations that can be measured in the CMB and agree remarkably well.
- There is hope that measuring tensor fluctuations and any deviation in the power spectrum can constrain different inflation models.

ANTHROPIC PRINCIPLE

- There are many possible scalar fields with many possible values for their potentials and no real justification for values that give us the Universe we see.

- One way around this difficult is to invoke an anthropic argument, the Universe we see must allow for our existence to see it.

- In this way of thinking the fine tuning of inflation is alleviated by assuming all potentials happen and then asking the question in which universe could we exist. This then favors one like we have because most universes have too much or too little matter, galaxies and stars don’t form, etc.

MORE HOMEWORK

- 2. What upper limit can be placed on the radiation density $\Omega_r(m_P)$ by requiring that the Universe not end in a Big Crunch before inflation assuming inflation starts at $t=10^{-36}$ s.

- 3. The measured vacuum energy density today is $\varepsilon_\Lambda = 0.7\ \varepsilon_{c,0} = 3600\ \text{MeV/m}^3$. What will be the the value of H once dark energy becomes strongly dominate?