THE THERMAL HISTORY OF THE UNIVERSE

Week 5

OUTLINE

- Photon Decoupling - CMB
- Neutron Decoupling - BBN
- Baryogensis
- WIMP Decoupling - CDM

THERMAL EQUALIBRIUM

- As the scale factor gets smaller the density of both matter and radiation increases.
- Thus the early Universe was hotter and denser than today. Unless there is a process whose cross section decreases with energy faster than the density of particles is increasing with scale factor, we expect all particles to be in thermal equilibrium in the early Universe.
- As the Universe cools and energies and densities decrease we expect particles to fall out of equilibrium when their interaction rates become low.

INTERACTION RATES

- The mean free path of a particle $\lambda = 1/n\sigma$.
- And the interaction rate $\Gamma = v/\lambda = vn\sigma$.
- Particles will remain coupled as long as the interaction rate is greater than the expansion rate.

$$\Gamma > H(z)$$

- For relativistic particles moving at speed c, this is the same as saying their mean free path is less than the Hubble distance, $\lambda < c/H$.

DISTRIBUTION FUCNTIONS

- If particles are in thermal equilibrium then statistical mechanics will apply and the energy and momentum of a species i will be given by its distribution function.

$$n = \frac{g_i}{(2\pi\hbar)^3}\int f(\vec{p})d^3\vec{p}$$

$$\epsilon = \frac{g_i}{(2\pi\hbar)^3}\int E(\vec{p})f(\vec{p})d^3\vec{p}$$

$$P = \frac{g_i}{(2\pi\hbar)^3}\int \frac{|\vec{p}|^2}{3E}f(\vec{p})d^3\vec{p}$$

- Where f(p) depends on if you have bosons are fermions.

$$f_b = \frac{1}{e^{kT-\mu}-1} \qquad f_f = \frac{1}{e^{kT-\mu}+1}$$

DISTRIBUTION FUNCTIONS

- In the limit of non-relativistic velocities, a negligible chemical potential μ, and quantum effects are negligible we get the Maxwell-Boltzmann distribution.

$$n_i = g_i \left(\frac{m_i kT}{2\pi\hbar^2} \right)^{\frac{3}{2}} e^{-\frac{m_i c^2}{kT}}$$

- If we consider for example the case of hydrogen versus ionized protons and neutrons we get

$$\frac{n_H}{n_p n_e} = \frac{g_H}{g_p g_e} \left(\frac{m_H}{m_p m_e} \right)^{\frac{3}{2}} \left(\frac{kT}{2\pi\hbar^2} \right)^{-\frac{3}{2}} e^{-\frac{(m_p + m_e - m_H)c^2}{kT}}$$

THE SAHA EQUTATION

- That equation can be simplified by setting $m_H/m_p=1$.
- Recognizing that $g_e=g_p=2$ and $g_H=4$.
- And that the difference in mass between hydrogen and a free proton and electron is the binding energy Q=13.6eV.
- Then one gets the Saha equation

$$\frac{n_H}{n_p n_e} = \left(\frac{m_e kT}{2\pi\hbar^2}\right)^{-\frac{3}{2}} e^{-\frac{Q}{kT}}$$

CROSS SECTION

- In a fully ionized gas scattering is dominated by the electrons. The scattering cross section is

$$\sigma_T = \frac{8\pi}{3} r_e^2 = \frac{e^4}{6\pi\epsilon_0^2 m_e^2 c^4}$$

- In neutral hydrogen the cross section becomes Rayleigh scattering,

$$\sigma = \frac{\nu^4}{(\nu^2 - \nu_0^2)^2}\sigma_T$$

- where ν_0 is the natural frequency of the atom (13.6eV or 91nm for hydrogen). We see that for $\nu << \nu_0$ the cross section drops dramatically.

$$\sigma = \frac{\nu^4}{\nu_0^4}\sigma_T$$

- So the cross section to photons dramatically changes for most photons when the Universe goes from being ionized to neutral. When did this occur?

- Well we could imagine it occurred when the peak of the blackbody curve was13.6 eV or ~32,000K.

- However, the photon energies are a distribution and if there are many more photons than electrons it might be that only 10% or 1% or 0.01% of the distribution is needed to ionize the Universe.

- This introduces a new important parameter the baryon-to-photon ratio.

BARYON-TO-PHOTON RATIO

- This baryon-to-photon ratio, η_γ, is the ratio of baryons to photons in the Universe.

- Current measurements find $\eta_\gamma=6.1\times10^{-10}$. This might seem rather small, but the real question is why is it so large.

- That is why is it not 0, where are there any baryons in the Universe. It is believed that this is the result of some interaction that caused baryon asymmetry in the early Universe.

- This is called baryogensis and points to physics beyond the standard model of particle physics.

DECOUPLING

- What happens to a particle that decouples and no longer has interactions.
- This is not the condition of thermal equilibrium, however the distribution function of the particle still remains the same with only an effective temperature changing.

BLACK BODY DISTRIBUTION

Let’s start with radiation that is in thermal equilibrium at some epoch, what will happen to it as the Universe expands.

$$\nu' = \frac{v}{1+z} \quad \text{and} \quad d\nu' = \frac{dv}{1+z}$$

The radiation will originally have a black body spectrum

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/k_BT}-1}$$

Dividing by the energy per photon E=hν, we get the number of photons in a narrow band dν.

$$n_\nu = \frac{2\nu^2}{c^2}\frac{d\nu}{e^{h\nu/k_BT}-1}$$

The number of photons is conserved as the Universe expands so

$$n'_\nu = \frac{n_\nu}{(1+z)^3} = \frac{2\nu^2}{c^2}\frac{d\nu}{e^{h\nu/k_BT}-1}\frac{1}{(1+z)^3} = \frac{2\nu'^2}{c^2}\frac{d\nu'}{e^{h\nu/k_BT'}-1}$$

where $T' \equiv T/(1+z)$

BLACK BODY RADIATION

- As the Universe expands thermal radiation will maintain the black body curve with a temperature that changes as $T(z) \propto (1+z)^{-1}$.
- This temperature is usually referred to as the temperature of the Universe, since many other particles will be in equilibrium with photons.

COSMIC MICROWAVE BACKGROUND

COSMIC MICROWAVE BACKGROUND

- Between 1946-48 Gamov and Dicke predicted that there should be a relic radiation from the last time matter and radiation where in thermal equilibrium with temperature between ~5-50K.

- This occurred at the time of recombination when the energy of photons decreased so that protons and electrons could form hydrogen atoms.

- Notice in standard astronomy speak this is called recombination even though it was the first time atoms combined.

- Once the free electrons disappear the cross section to scattering drops and photons decouple from matter.

DISCOVERY

- In the 1960's at Princeton, Dicke, Peebles and Wilkinson design an experiment to look for this microwave background.

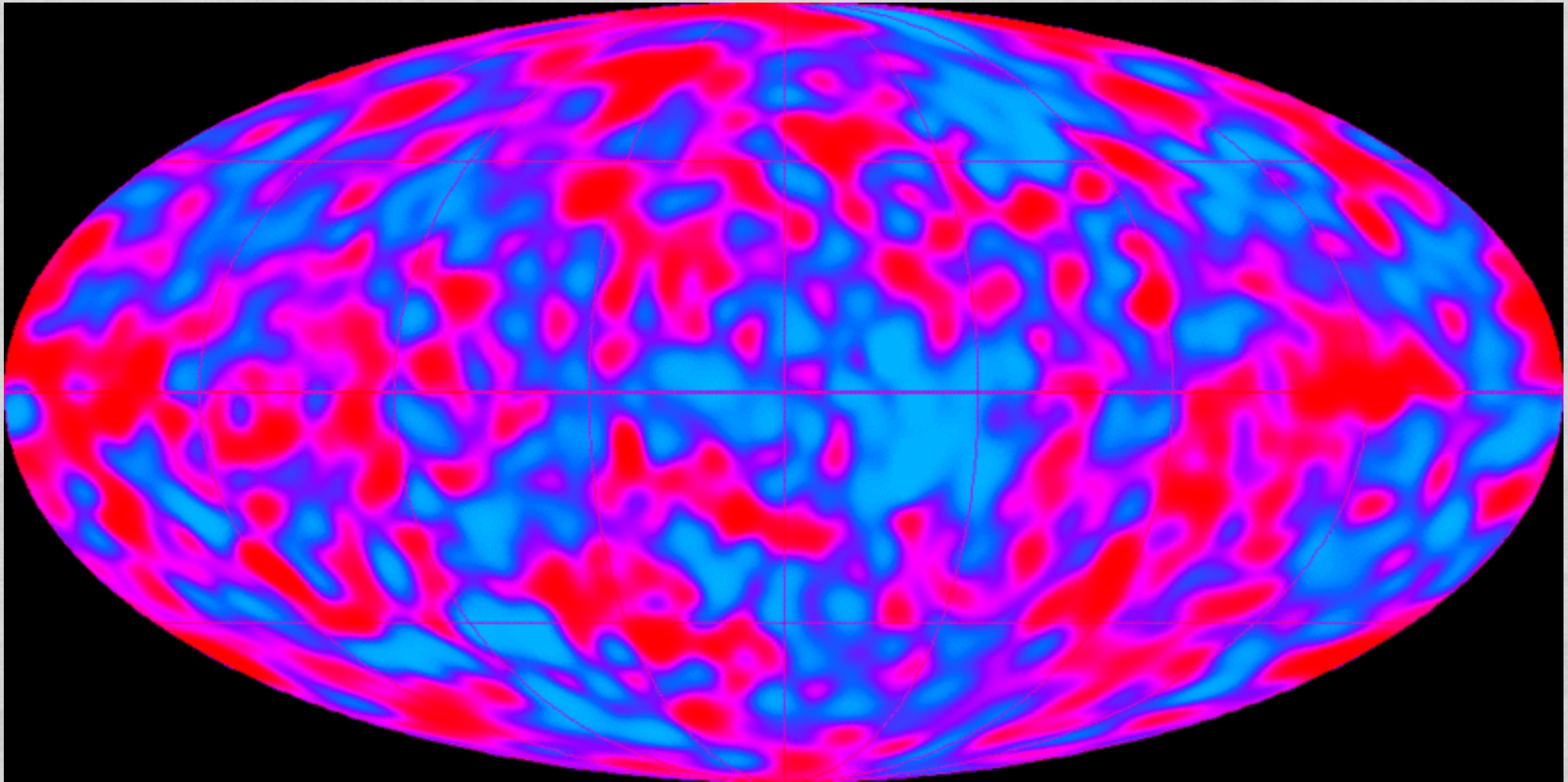
- At the same time Penzias and Wilson, working on a very sensitive radio receiver at 7.35mm were trying to eliminate all sources of noise.

- However, they found a persistent signal 100x what they expected that was there all the time from every direction.

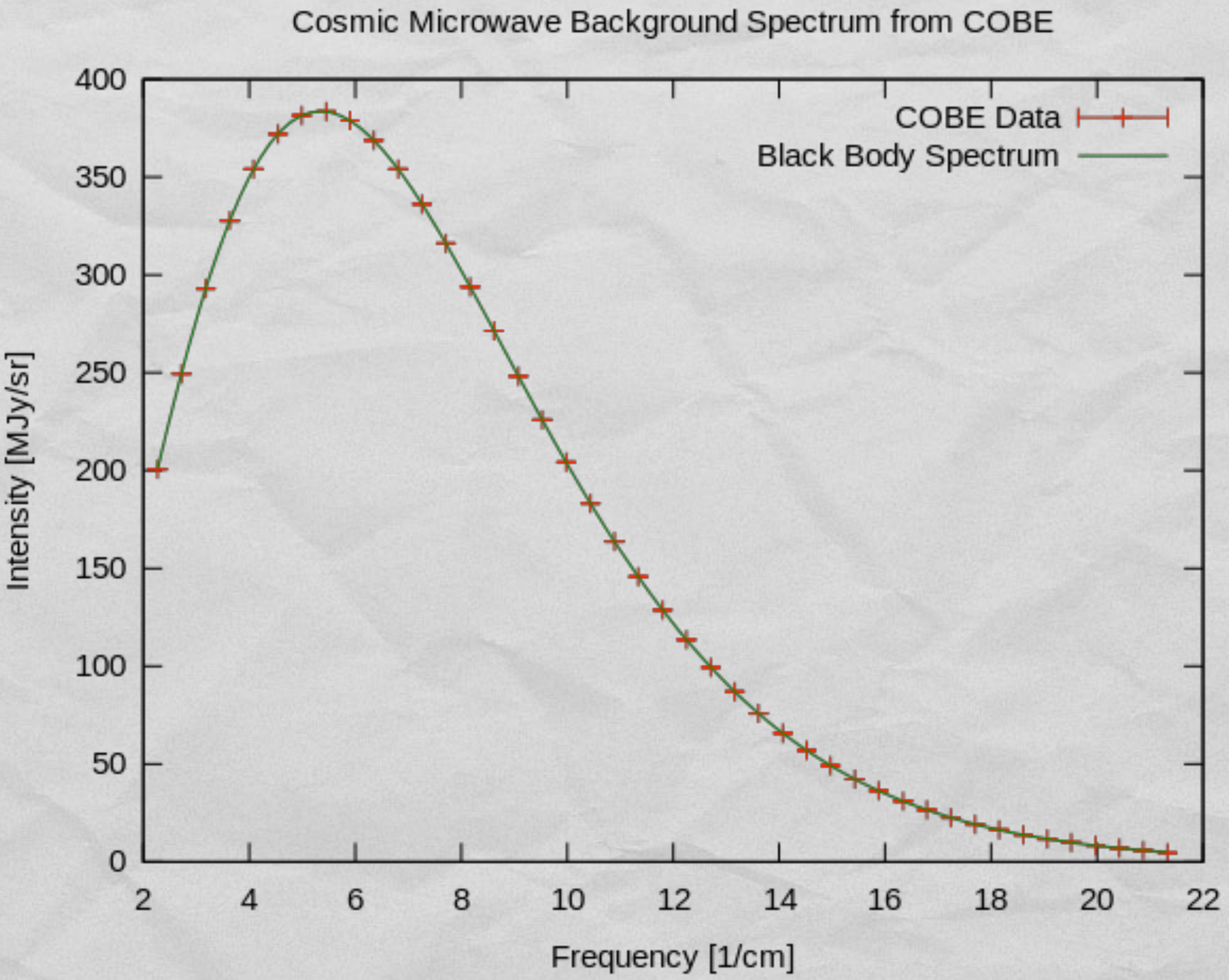
- A mutual friend put Penzias and Dicke in contact and they realized that the signal Penzias and Wilson found was the CMB. They won the Nobel prize for this in 1978.

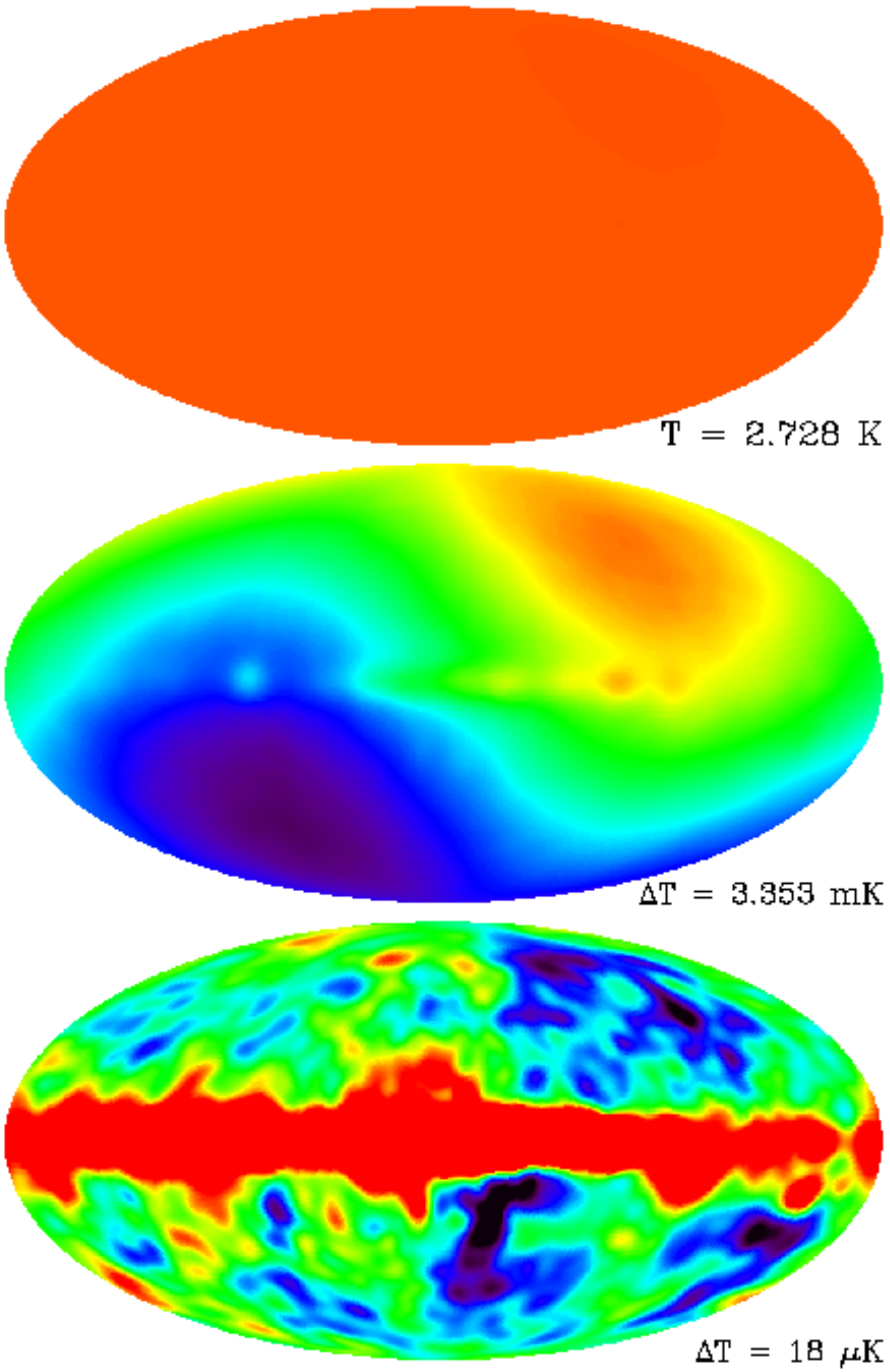
COBE

COBE (the Cosmic Background Explorer) was the satellite launched by NASA in 1989 to study the CMB and measure its fluctuations.



COBE found that the CMB was essentially a perfect black body, a better black body than any measured in a lab. This strongly suggests its cosmological origin is correct.





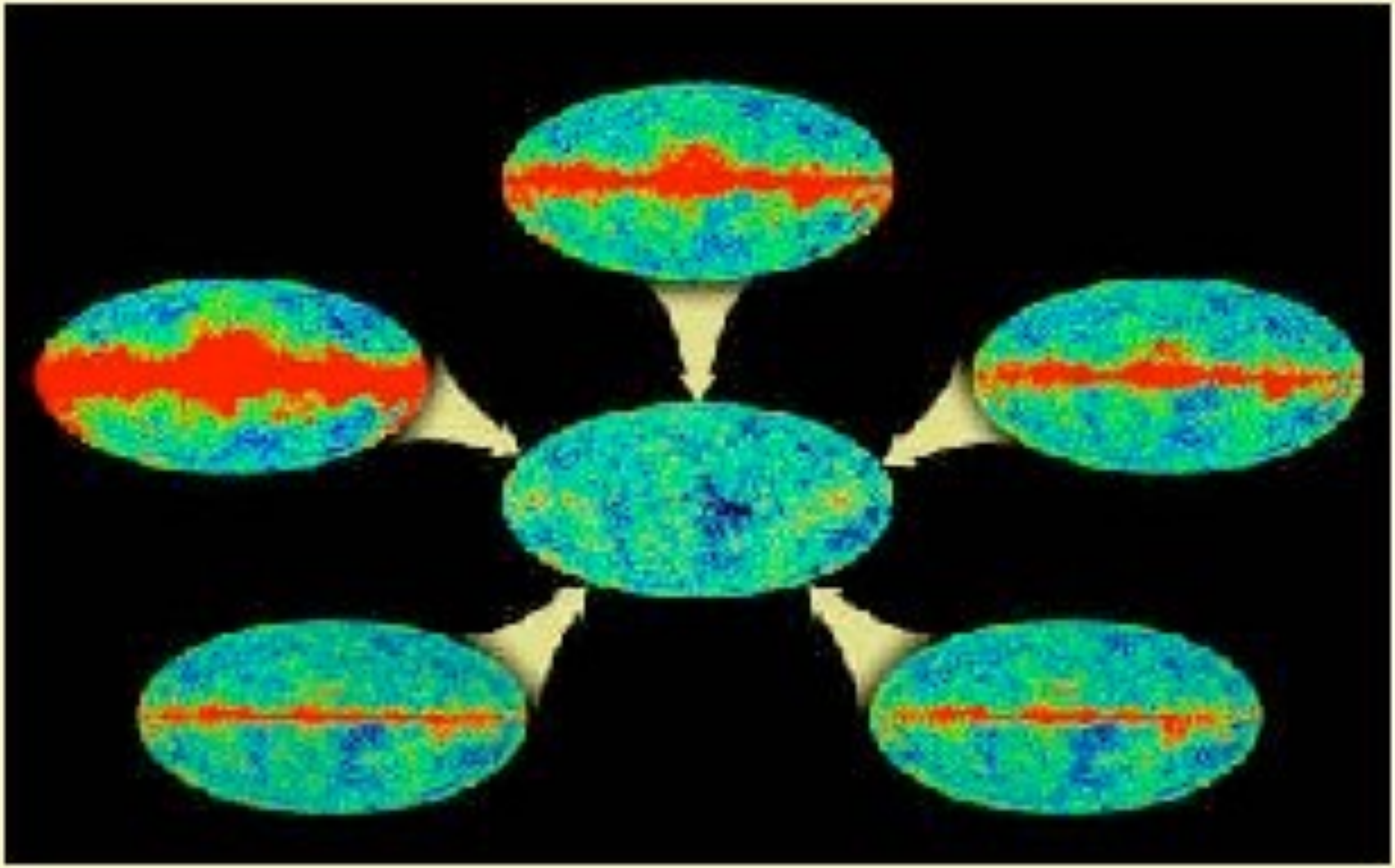
The CMB is a perfect uniform blackbody at T=2.728K. One has to get to mK before spatial differences are seen.

At a few mK the CMB shows a strong dipole due to the doppler shift of the Earth’s motion in the Universe. The CMB means there is a preferred velocity frame.

One has to reach μK before fluctuations in the CMB are seen. These fluctuations are the seeds of all structure formation.

WMAP

- The Wilkinson Microwave Anisotropy Probe (WMAP) was launched in 2001.
- Mission goal was to measure cosmological parameters from temperature fluctuations to 1%.
- Also measures polarization.
- Has 5 frequency channels to remove foreground objects.

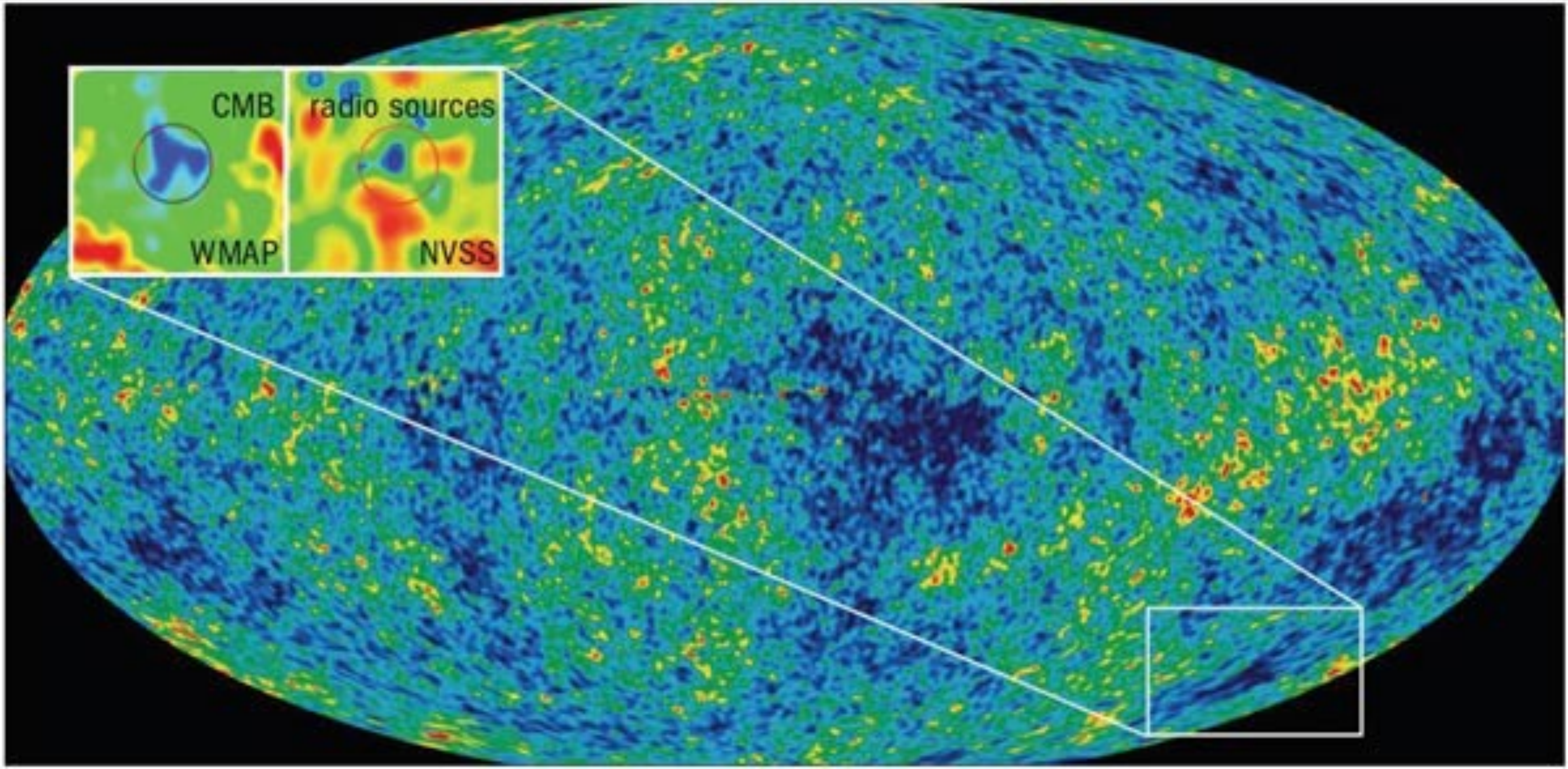


Data is taken in 5 frequency bands, foregrounds can be identified by not having a 2.7K blackbody spectrum.

ANOMOLIES

- WMAP has a few unusual aspects that are the subject of much research because they may imply new physics.
- The quadruple moment of the map is somewhat lower than models predict (~1.5σ).
- There is a cold spot that is 70μK below the mean temperature (where 18μK is the rms) and subtends 5°.

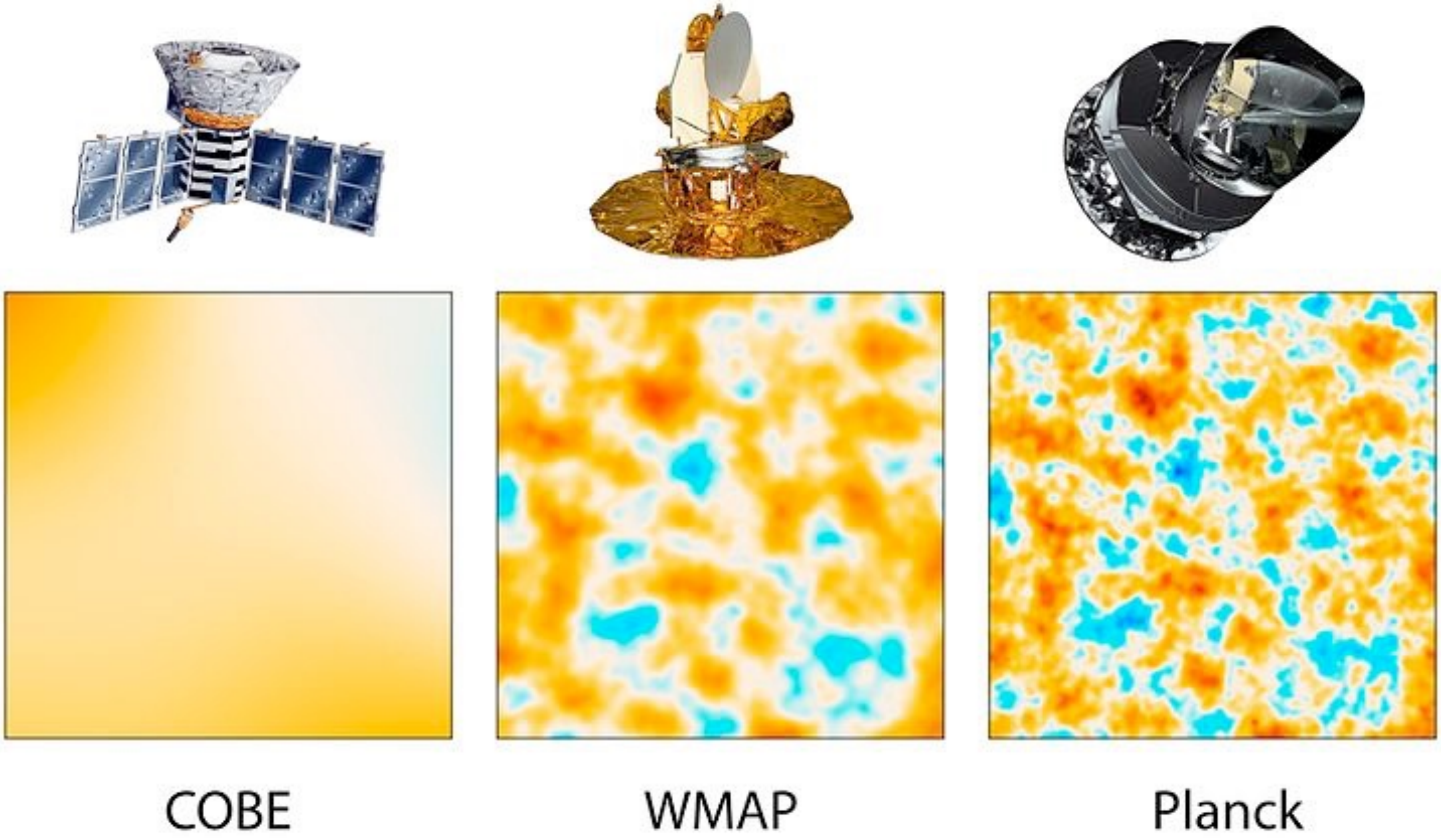
WMAP COLD SPOT

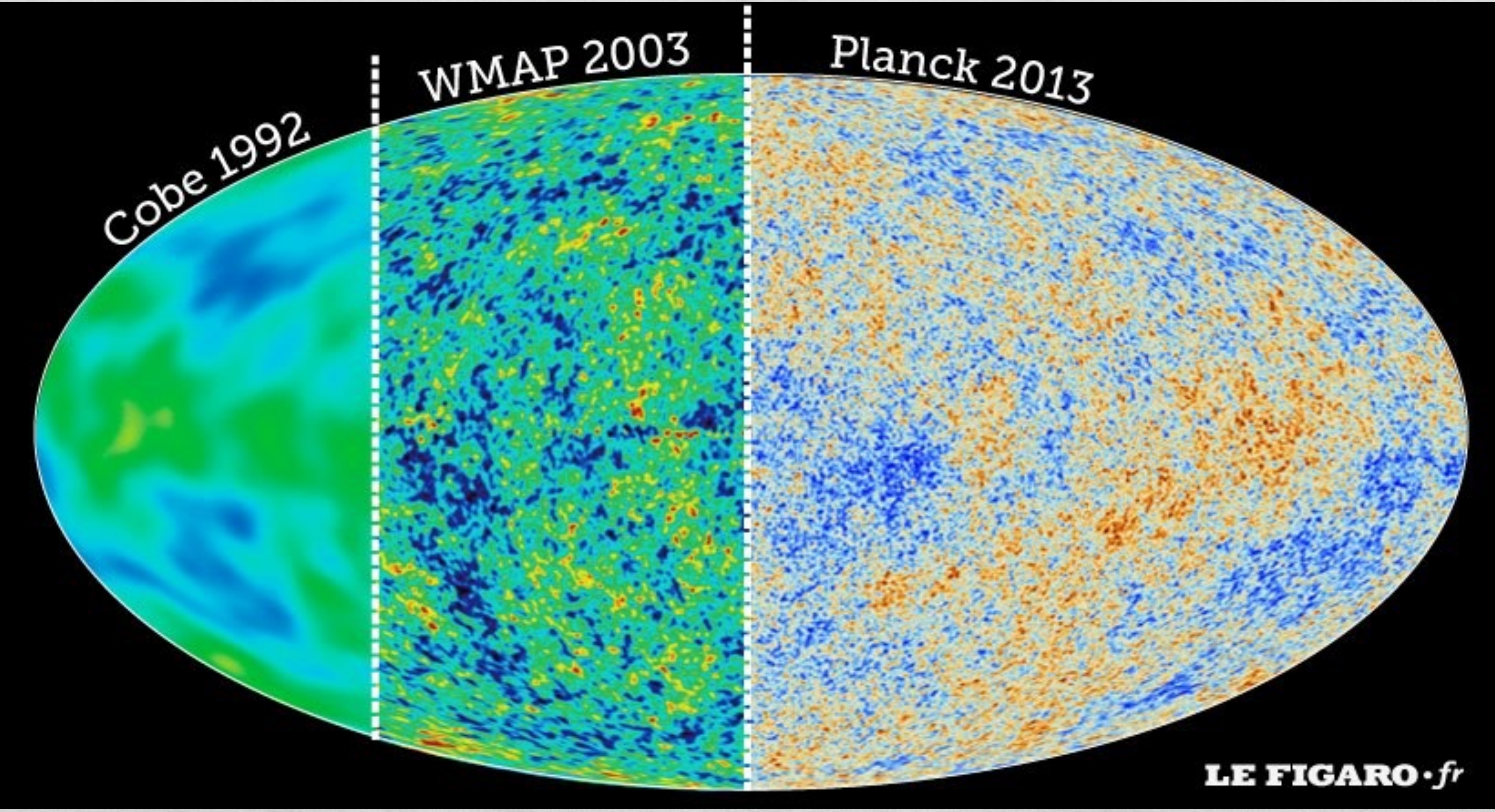


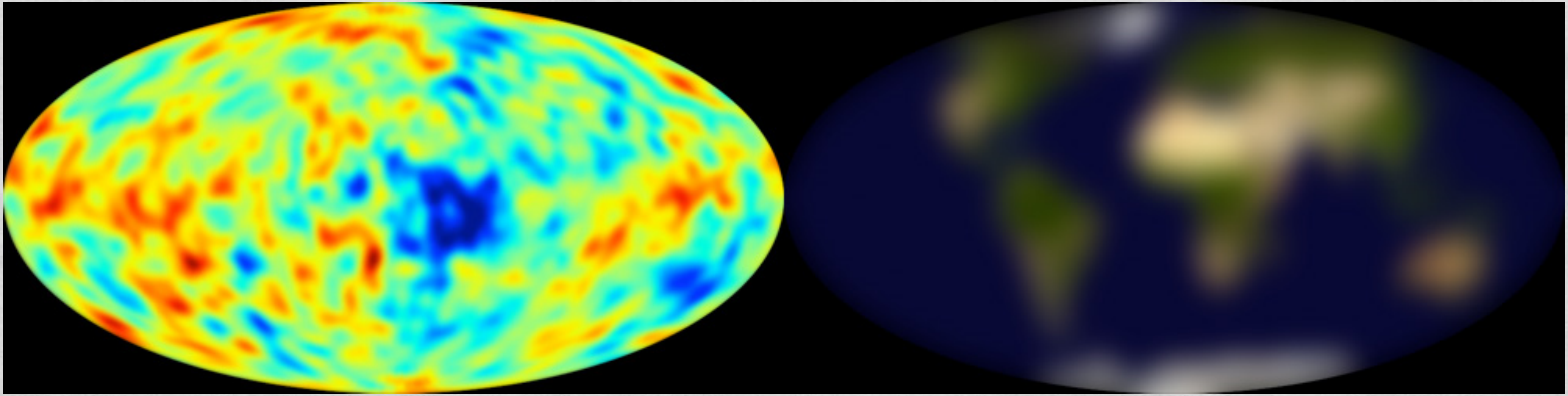
PLANK

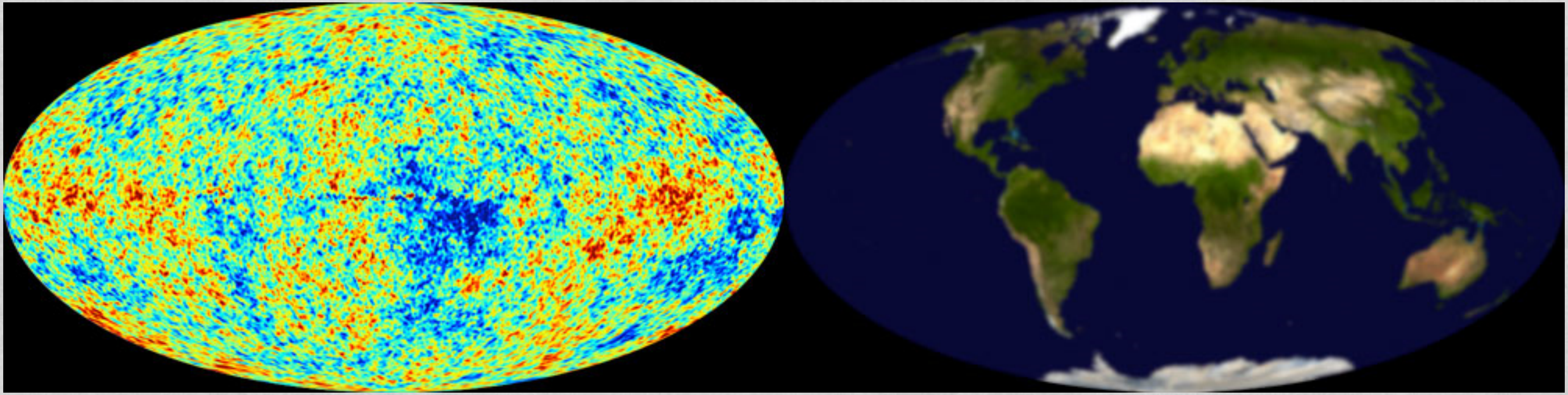
- Plank was launched by ESA in 2009.
- 9 frequency channels.
- 3x better resolution that WMAP

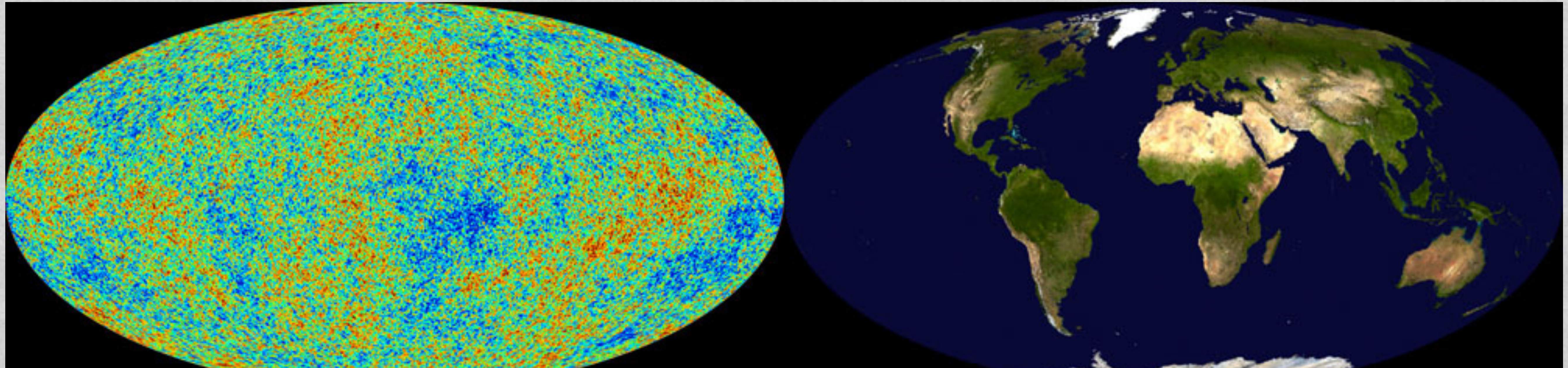
SATELLITE COMPARISON

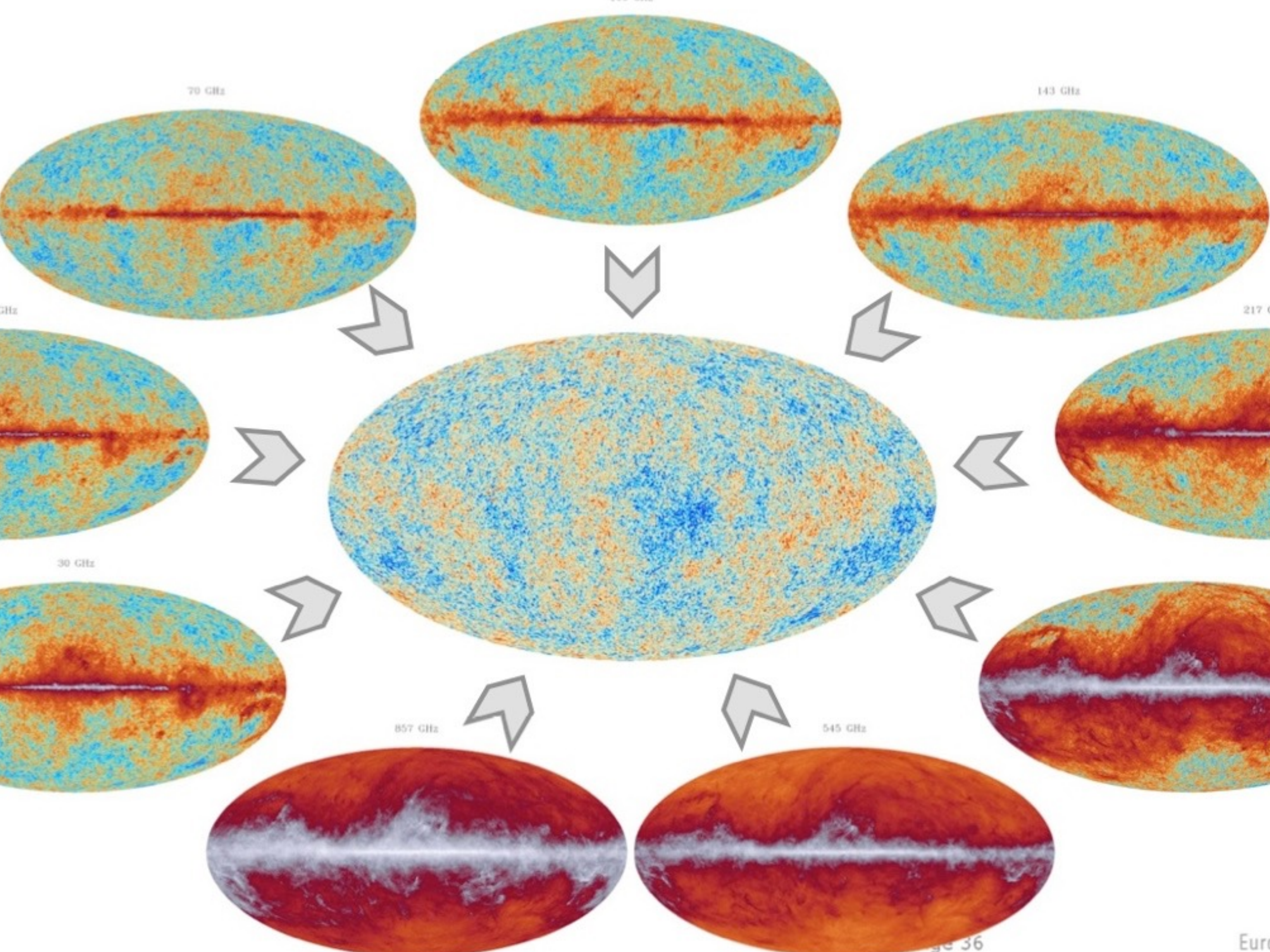



Cobe 1992
WMAP 2003
Planck 2013
LE FIGARO·fr








70 GHz
143 GHz
GHz
217
30 GHz
857 GHz
545 GHz

BALLON/ROCKET EXPERIMENTS

- Note that there have also been many ballon and rocket experiments that have measured the CMB often beating the satellites to important results.
- However, since these are not full sky they always have less statistics.
- With no current plans for another CMB satellite ground based or ballon based experiments are where current research is most active.

FLUCTUATIONS

- The fluctuations in the CMB show that we do not live in a homogenous universe.
- However, the size of the fluctuations show that the Universe was very very close to homogenous especially at early times.
- Thus consideration of the Robertson-Walker metric is very close to the real thing and one can study the Universe as perturbations on the Robertson-Walker metric.

PHOTON NUMBER DENSITY

The energy density at a given frequency is given by the blackbody

$$S_\nu = \frac{8\pi h}{c^3}\frac{\nu^3}{e^{h\nu/kT}-1}$$

So dividing by the energy of a photon E=hν gives the number density

$$n_\nu = \frac{8\pi}{c^3}\frac{\nu^2 d\nu}{e^{h\nu/kT}-1}$$

So the total photon density we can get by integrating

$$n = \int_0^\infty n_\lambda d\lambda = \int_0^\infty \frac{8\pi}{c^3}\frac{\nu^2 d\nu}{e^{h\nu/kT}-1} = 16\pi\zeta(3)(\frac{kT}{hc})^3$$

BIG BANG NUCLEOSYNTHESIS

NEUTRON FREEZEOUT

- The rest mass energy difference between the neutron and proton is $(m_n - m_p)c^2 = 1.3\,\mathrm{MeV}$.
- So at $t < 1\,\mathrm{s}$ when the temperature was $T > 1\,\mathrm{MeV}$ (10^{10} K) the following reactions could occur:

$$e + p \leftrightharpoons \nu_e + n$$

$$\nu_e + p \leftrightharpoons e^+ + n$$

- So we would expect neutrons and protons to be in thermal equilibrium.

NEUTRON FREEZEOUT

The ratio of neutron to protons will be given by

$$\frac{N_n}{N_p} = \left(\frac{m_n}{m_p}\right)^2 e^{-\frac{(m_n - m_p)c^2}{kT}}$$

- At high temperatures the ratio of neutrons to protons is close to one.

- As the Universe cools, the ratio decreases.

- But at $T<0.8$MeV the mean time for electron-proton combination becomes longer than the age of the Universe (2s) and neutrons decouple from the protons.

- The neutrons freeze at a ratio of $e^{(1.3/0.8)}=0.20$. In the following few minutes most of the neutrons become bound up in Helium nuclei.

DEUTERIUM

- Neutrons have a half live of 15 minutes, so if those neutron stayed free they would all have be gone a long long time ago.
- But neutrons in atomic nuclei can be stable. So all the neutrons today are the ones that fused with protons to form nuclei.
- There are 3 reactions that from deuterium but only one is relevant, $p+n \leftrightharpoons e+\gamma$, because the other two are weak interactions.

$$p + n \leftrightharpoons D + e$$

$$n + n \leftrightharpoons D + e + \bar{\nu}_e$$

$$p + p \leftrightharpoons D + e^+ + \nu_e$$

Note that the last one is how fusion occurs in stars, but it is a very slow process. A proton in the Sun's core has only a 10^{-10} chance of undergoing fusion this year.

LIGHT ELEMENTS

- Deuterium is stable so once the neutrons are bound into deuterium they are safe.
- However the Universe is still very hot and dense, other nuclear reactions can occur.
- Deuterium will fuse to form ^{3}H and ^{3}He both of which will fuse to form ^{4}He.
- These are all strong interaction and proceed fairly quickly.

LIGHT ELEMENTS

- Once you get to ^{4}He however you run into a bottle neck. There are no stable nuclei with 5 nucleons. You have to jump to ^{6}Li or ^{7}Li. These can be formed by the following reactions.

$$^4He + D \leftrightharpoons {}^6Li + \gamma$$

$$^4He + {}^3H \leftrightharpoons {}^7Li + \gamma$$

- These reactions are going to be much less common because the abundance of D and ^{3}H are much less than protons.
- In addition ^{7}Be can be formed via

$$^4He + {}^3He \leftrightharpoons {}^7Be + \gamma$$

LIGHT ELEMENTS

- Reaching 8 nucleons again causes a problem. ^{8}Be will decay into two ^{4}He with a half life of 3×10^{-16}s.
- Thus the vast majority of neutrons are bound in ^{4}He. Trace amounts of D, ^{3}He, ^{6}Li, ^{7}Li and ^{7}Be are formed and essentially nothing with $A > 7$.
- Thus the abundances of light elements in the pristine Universe is a direct prediction of Big Bang Nucleosynthesis.

HELIUM FRACTION

- If we ignore other nuclei and assume all the neutrons end up in ^{4}He then we would have 6 protons for every ^{4}He.
- The mass fraction of He in the Universe would then be

$$Y_{He} = \frac{4N_{He}}{N_H + 4N_{He}} = \frac{4(1)}{6+4(1)} = 0.4$$

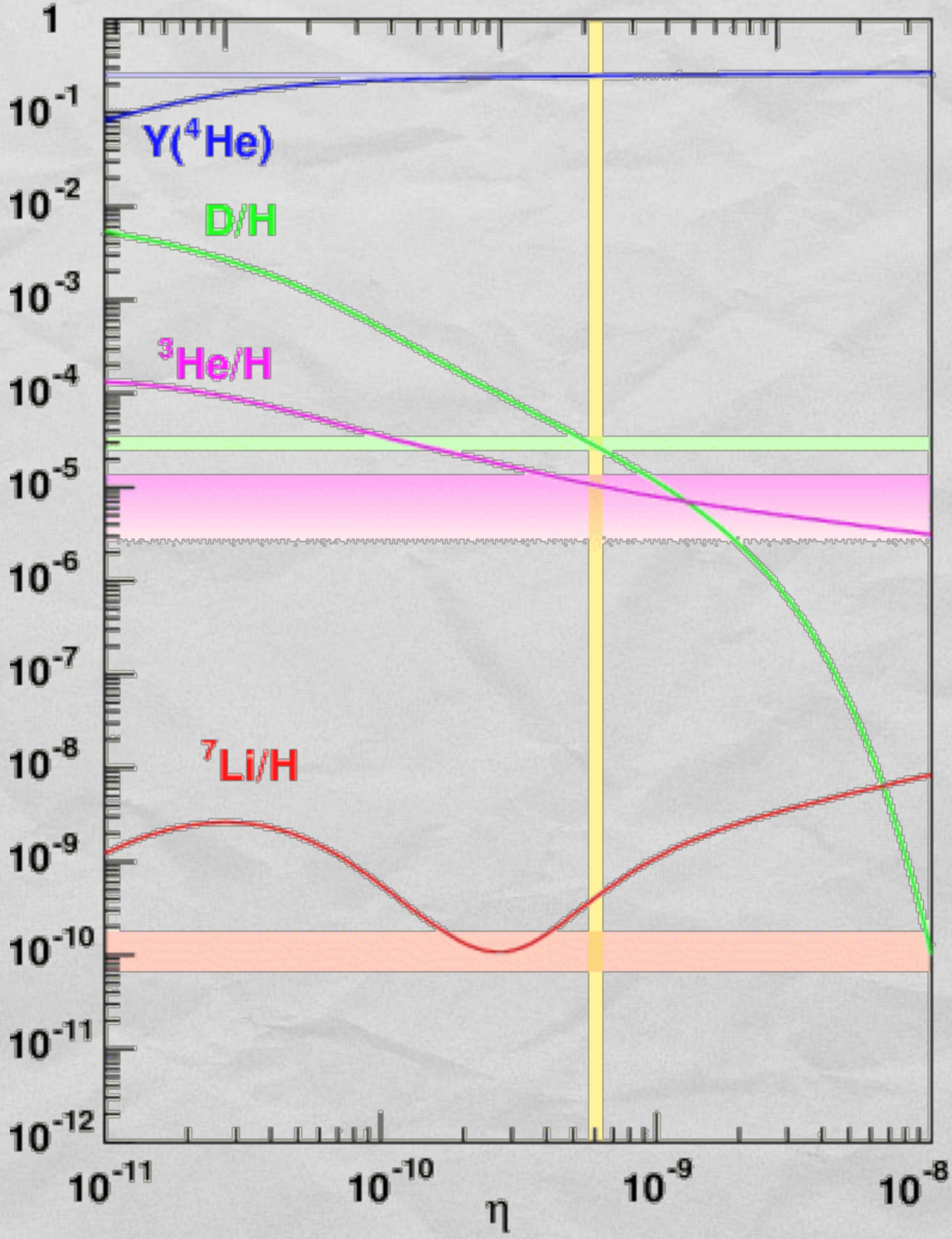
- It turns out both because some neutrons decay and some are in other nuclei that the ratio of ^{4}He to protons is 1:7 instead of 1:5 so Y_{He} becomes

$$Y_{He} = \frac{4(1)}{12+4(1)} = 0.25$$

Performing all the nuclear reaction calculations one can determine the relative abundance of each species.

The only parameter is the baryon to photon ratio. Attempts to measure the abundances of these elements in pristine environments give very good agreement with calculations.

The resulting value of η is also in good agreement with its value measured from the CMB. The small discrepancy in Li still needs to be explained.



BARYOGENSIS

BARYON-ANTIBARYON ASYMMETRY

- As we noted earlier the baryon-to-photon ratio is a very small number, but not zero.
- That may simply reflect the original baryon number in the Universe.
- However, it has been proposed that if an interaction exists that is asymmetric in baryon-antibaryon number then that would explain the presence of baryons today.
- Why there are few, but not zero baryon in the Universe.

BARYON-ANTIBARYON ASYMMETRY

- This would have to happen at t < 2s to set the photon-to-baryon ratio that we find in Big Bang Nucleosynthesis.
- We imagine that at high enough temperature baryons and antibaryons are in equilibrium. But there is some as yet unknown physics that creates a small energy difference for at least one baryon/antibaryon pair.
- When the Universe has a temperature at that energy scale slightly more baryons are produced then anti baryons. Then dropping below that energy scale the particles get frozen out and the asymmetry remains today.

WIMP DECOUPLING

- It is also possible to understand the origin of cold dark matter from decoupling.

- We have examined how photons and neutrons decouple as their interaction rate becomes longer than the Hubble rate.

- The same thing could have happened early on to other particles.

- If a particle of mass ~100GeV decouples at the weak scale then the mass density today would be of order the critical density.

- This was the original motivation for CDM. It also places constraints on any new particle physics. At some temperature the particle was in thermal equilibrium with the rest of the Universe so we can calculate the number density of that particle after decoupling.

- Note most particles decay so this is only relevant for particles that can't decay into standard model particles.

PROBLEMS

- Estimate the temperature of neutrino decoupling by setting the reaction rate equal to the Hubble constant. You can assume the Universe is dominated by photons, the neutrino-photon cross section is $10^{-43}\mathrm{cm}^2(E_\nu/1\mathrm{MeV})^2$, the photon density is $\sigma_{SB}T^4/kT$ and the neutrino energy is kT.